ASTRO 358 : Quiz 1, Solution Set

Total score = 80 points.

1(a) Each fusion equation, starting with the mass M_1 of 4 protons ($4 \times 1.6726 \times 10^{-27}$ kg) and ending with the mass M_2 (6.643×10^{-27} kg) of a helium nucleus generates an amount of energy E, given by

$$E = (M_1 - M_2)\ c^2 = (4.7 \times 10^{-29}\text{kg})\ \text{c}^2 = \text{f M}_1\ \text{c}^2 = 4.2 \times 10^{-12}\text{J} \tag{1}$$

where the efficiency factor $f = 0.0067$ [**10 pts**].

1(b) The mass$M$ of hydrogen fused per second by the Sun to produce a luminosity of $L = 4 \times 10^{26}$ W is given by [**10 pts**]:

$$M = \frac{L}{f\ c^2} = \frac{4 \times 10^{26}}{0.0067\ c^2} = 6.6 \times 10^{11}\text{kg} \tag{2}$$

2. Iron is the most tightly bound element and hence its fusion into any heavier element will absorb energy rather than release it. [Another equivalent answer is: Fusion only releases energy if the end-product is of lower effective mass than the starting fuel. However, iron has the smallest effective mass per nuclear particle among all elements, and its fusion can only lead to a product of larger effective mass, thereby absorbing energy.] [**10 pts**].

3. For the stellar mass range $M = 2 - 10 M_\odot$, the main sequence lifetime scales as

$$T_{\text{ms}} \propto M^{1-n} \tag{3}$$

where $n \sim 3.5$. Hence the lifetime of the $10 M_\odot$ stars is $\sim$ 10Gyr $\times\ (10/2)^{-2.5}$ or ~ 0.17 Gyr. [**10 pts**]

4. For a low mass ($M \sim\ 1\ M_\odot$) star: the outer layers become a planetary nebula, while the inert core becomes a white dwarf (supported by electron degeneracy pressure). ([**10 pts**]).

5. For a high mass ($M \sim\ 10\ M_\odot$) star: the outer envelope is ejected as a supernova remnant, and the inert iron core generally implodes into a neutron star (supported by neutron degeneracy pressure.) [Extra comment: In an extreme case, where the core mass is larger than the Tolman-Oppenheimer-Volkoff limit (1.5 to 3 $M_\odot$) it will become a black hole, but this typically happens only in very high mass ($M >\ 35\ M_\odot$) stars.] ([**10 pts**]).

6. Stefan-Boltzmann law : The total flux F_{surf} emitted at the surface of a star (or blackbody) over all wavelengths is proportional to the fourth power of its surface temperature T:

$$F_{\text{surf}} = \sigma T^4 \tag{4}$$

where σ = Stefan-Boltzmann constant = 5.7×10^{-8} J s^{-1} m^{-2} K^{-4}

Flux at surface of a star of radius R:

$$F = \frac{L}{4\pi R^2} \tag{5}$$

Hence,

$$L = 4\pi R^2 \sigma T^4 \propto R^2 T^4 \tag{6}$$

Flux that we receive at distance d

$$f = \frac{L}{4\pi d^2} \propto \frac{R^2 T^4}{d^2} \tag{7}$$

If the radius and surface temperature of a star both double, the flux that we receive rises by a factor of 4x16 or 64.

[[**10 pts**] = 7 pts for $f \propto (R^2\ T^4/d^2)$ + 3 pts for math.]

7. B-K = m_{B} - m_{K} = -2.5 log($f_{\mathrm{B}}/\ f_{\mathrm{K}}$) = 3.2 mag.

If the flux of the galaxy at blue wavelengths rises by a factor of 100, the B-K color falls by 5 magnitudes to -1.8 mag.

[[**10 pts**]= 6 pts for showing relation of color to flux + 4 pts for math.]