

## 1.1 The stars

### 1.1.1 Star light, star bright ...

All the information we have about stars more distant than the Sun has been deduced by observing their electromagnetic radiation, mainly in the ultraviolet, visible, and infrared parts of the spectrum. The light that a star emits is determined largely by its surface area, and by the temperature and chemical composition – the relative numbers of each type of atom – of its outer layers. Less directly, we learn about the star's mass, its age, and the composition of its interior, because these factors control the conditions at its surface. As we decode and interpret the messages brought to us by starlight, knowledge gained in laboratories on Earth about the properties of matter and radiation forms the basis for our theory of stellar structure.

The *luminosity* of a star is the amount of energy it emits per second, measured in watts, or ergs per second. Its *apparent brightness* or *flux* is the total energy received per second on each square meter (or square centimeter) of the observer's telescope; the units are  $\text{W m}^{-2}$ , or  $\text{erg s}^{-1} \text{cm}^{-2}$ . If a star shines with equal brightness in all directions, we can use the *inverse-square law* to estimate its luminosity  $L$  from the distance  $d$  and measured flux  $F$ :

$$F = \frac{L}{4\pi d^2}. \quad (1.1)$$

Often, we do not know the distance  $d$  very well, and must remember in subsequent calculations that our estimated luminosity  $L$  is proportional to  $d^2$ . The Sun's total or *bolometric* luminosity is  $L_{\odot} = 3.86 \times 10^{26} \text{ W}$ , or  $3.86 \times 10^{33} \text{ erg s}^{-1}$ . Stars differ enormously in their luminosity: the brightest are over a million times more luminous than the Sun, while we observe stars as faint as  $10^{-4} L_{\odot}$ .

Lengths in astronomy are usually measured using the *small-angle* formula. If, for example, two stars in a binary pair at distance  $d$  from us appear separated on the sky by an angle  $\alpha$ , the distance  $D$  between the stars is given by

$$\alpha \text{ (in radians)} = D/d. \quad (1.2)$$

Usually we measure the angle  $\alpha$  in *arcseconds*: one arcsecond ( $1''$ ) is 1/60 of an arcminute ( $1'$ ) which is 1/60 of a degree. Length is often given in terms of the *astronomical unit*, Earth's mean orbital radius (1 AU is about 150 million kilometers) or in *parsecs*, defined so that, when  $D = 1 \text{ AU}$  and  $\alpha = 1''$ ,  $d = 1 \text{ pc} = 3.09 \times 10^{13} \text{ km}$  or 3.26 light-years.

The orbit of two stars around each other can allow us to determine their masses. If the two stars are clearly separated on the sky, we use Equation 1.2 to measure the distance between them. We find the speed of the stars as they orbit each other from the *Doppler shift* of lines in their spectra; see Section 1.2. Newton's equation for

the gravitational force, in Section 3.1, then gives us the masses. The Sun's mass, as determined from the orbit of the Earth and other planets, is  $\mathcal{M}_\odot = 2 \times 10^{30}$  kg, or  $2 \times 10^{33}$  g.

Stellar masses cover a much smaller range than their luminosities. The most massive stars are around  $100\mathcal{M}_\odot$ . A star is a nuclear-fusion reactor, and a ball of gas more massive than this would burn so violently as to blow itself apart in short order. The least massive stars are about  $0.075\mathcal{M}_\odot$ . A smaller object would never become hot enough at its center to start the main fusion reaction of a star's life, turning hydrogen into helium.

**Problem 1.1** Show that the Sun produces 10 000 times less energy per unit mass than an average human giving out about  $1 \text{ W kg}^{-1}$ .

The radii of stars are hard to measure directly. The Sun's radius  $R_\odot = 6.96 \times 10^5$  km, but no other star appears as a disk when seen from Earth with a normal telescope. Even the largest stars subtend an angle of only about  $0.05''$ ,  $1/20$  of an arcsecond. With difficulty we can measure the radii of nearby stars with an interferometer; in eclipsing binaries we can estimate the radii of the two stars by measuring the size of the orbit and the duration of the eclipses. The largest stars, the red supergiants, have radii about 1000 times larger than the Sun, while the smallest stars that are still actively burning nuclear fuel have radii around  $0.1R_\odot$ .

A star is a dense ball of hot gas, and its spectrum is approximately that of a *blackbody* with a temperature ranging from just below 3000 K up to 100 000 K, modified by the absorption and emission of atoms and molecules in the star's outer layers or *atmosphere*. A blackbody is an ideal radiator or perfect absorber. At temperature  $T$ , the luminosity  $L$  of a blackbody of radius  $R$  is given by the *Stefan–Boltzmann* equation:

$$L = 4\pi R^2 \sigma_{\text{SB}} T^4, \quad (1.3)$$

where the constant  $\sigma_{\text{SB}} = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ . For a star of luminosity  $L$  and radius  $R$ , we define an *effective temperature*  $T_{\text{eff}}$  as the temperature of a blackbody with the same radius, which emits the same total energy. This temperature is generally close to the average for gas at the star's 'surface', the *photosphere*. This is the layer from which light can escape into space. The Sun's effective temperature is  $T_{\text{eff}} \approx 5780 \text{ K}$ .

**Problem 1.2** Use Equation 1.3 to estimate the solar radius  $R_\odot$  from its luminosity and effective temperature. Show that the gravitational acceleration  $g$  at the surface is about 30 times larger than that on Earth.

**Problem 1.3** The red supergiant star Betelgeuse in the constellation Orion has  $T_{\text{eff}} \approx 3500 \text{ K}$  and a diameter of  $0.045''$ . Assuming that it is  $140 \text{ pc}$  from us, show that its radius  $R \approx 700R_{\odot}$ , and that its luminosity  $L \approx 10^5 L_{\odot}$ .

Generally we do not measure all the light emitted from a star, but only what arrives in a given interval of wavelength or frequency. We define the *flux per unit wavelength*  $F_{\lambda}$  by setting  $F_{\lambda}(\lambda)\Delta\lambda$  to be the energy of the light received between wavelengths  $\lambda$  and  $\lambda + \Delta\lambda$ . Because its size is well matched to the typical accuracy of their measurements, optical astronomers generally measure wavelength in units named after the nineteenth-century spectroscopist Anders Ångström:  $1 \text{ \AA} = 10^{-8} \text{ cm}$  or  $10^{-10} \text{ m}$ . The flux  $F_{\lambda}$  has units of  $\text{W m}^{-2} \text{ \AA}^{-1}$  or  $\text{erg s}^{-1} \text{ cm}^{-2} \text{ \AA}^{-1}$ . The *flux per unit frequency*  $F_{\nu}$  is defined similarly: the energy received between frequencies  $\nu$  and  $\nu + \Delta\nu$  is  $F_{\nu}(\nu)\Delta\nu$ , so that  $F_{\lambda} = (\nu^2/c)F_{\nu}$ . Radio astronomers normally measure  $F_{\nu}$  in *janskys*:  $1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$ . The apparent brightness  $F$  is the integral over all frequencies or wavelengths:

$$F \equiv \int_0^{\infty} F_{\nu}(\nu) d\nu = \int_0^{\infty} F_{\lambda}(\lambda) d\lambda. \quad (1.4)$$

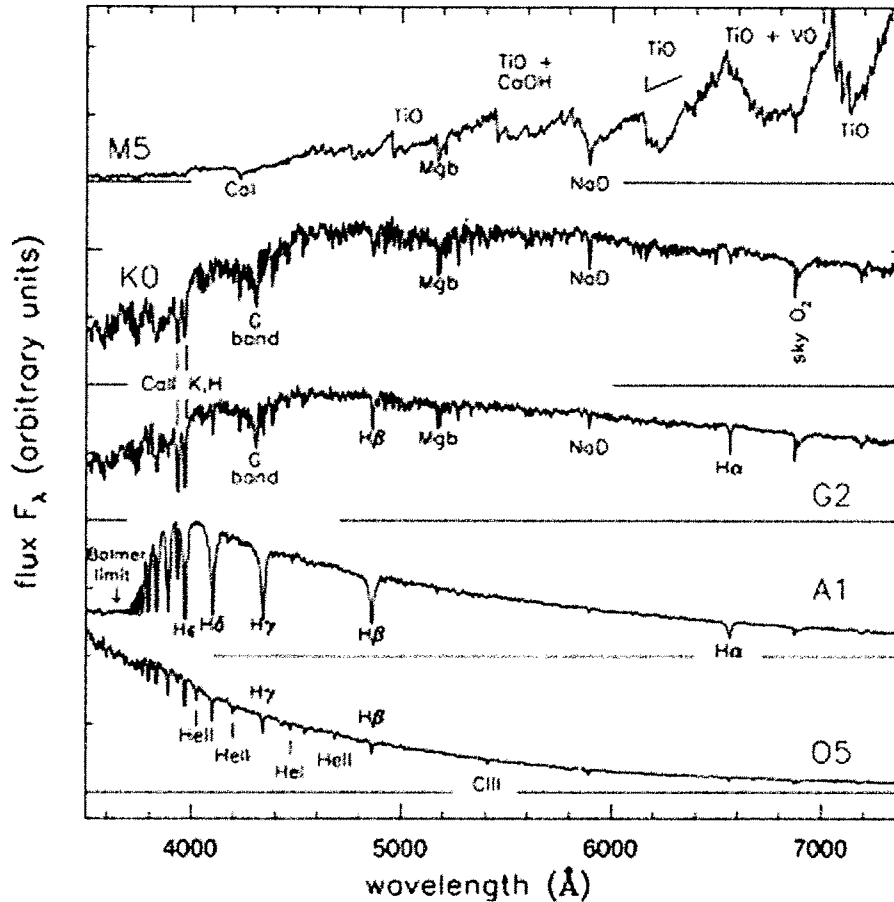
The hotter a blackbody is, the bluer its light: at temperature  $T$ , the peak of  $F_{\lambda}$  occurs at wavelength

$$\lambda_{\text{max}} = [2.9/T \text{ (K)}] \text{ mm}. \quad (1.5)$$

For the Sun, this corresponds to yellow light, at about  $5000 \text{ \AA}$ ; human bodies, the Earth's atmosphere, and the uncooled parts of a telescope radiate mainly in the infrared, at about  $10 \text{ }\mu\text{m}$ .

### 1.1.2 Stellar spectra

Figure 1.1 shows  $F_{\lambda}$  for a number of commonly observed types of star, arranged in order from coolest to hottest. The hottest stars are the bluest, and their spectra show absorption lines of highly ionized atoms; cool stars emit most of their light at red or infrared wavelengths, and have absorption lines of neutral atoms or molecules. Astronomers in the nineteenth century classified the stars according to the strength of the *Balmer* lines of neutral hydrogen HI, with A stars having the strongest lines, B stars the next strongest, and so on; many of the classes subsequently fell into disuse. In the 1880s, Antonia Maury at Harvard realized that, when the classes were arranged in the order O B A F G K M, the strengths of all the spectral lines, not just those of hydrogen, changed continuously along the sequence. The first large-scale classification was made at Harvard College Observatory between 1911 and 1949: almost 400 000 stars were included in the *Henry Draper Catalogue* and its supplements. We now know that Maury's spectral sequence lists the stars in order of decreasing surface temperature. Each of the classes has been subdivided



**Fig. 1.1.** Optical spectra of main-sequence stars with roughly the solar chemical composition. From the top in order of increasing surface temperature, the stars have spectral classes M5, K0, G2, A1, and O5 – G. Jacoby *et al.*, spectral library.

into subclasses, from 0, the hottest, to 9, the coolest: our Sun is a G2 star. Recently classes L and T have been added to the system, for the very cool stars discovered by infrared observers. Astronomers often call stars at the beginning of this sequence ‘early types’, while those toward the end are ‘late types’.

The temperatures of O stars exceed 30 000 K. Figure 1.1 shows that the strongest lines are those of HeII (once-ionized helium) and CIII (twice-ionized carbon); the Balmer lines of hydrogen are relatively weak because hydrogen is almost totally ionized. The spectra of B stars, which are cooler, have stronger hydrogen lines, together with lines of neutral helium, HeI. The A stars, with temperatures below 11 000 K, are cool enough that the hydrogen in their atmospheres is largely neutral; they have the strongest Balmer lines, and lines of singly ionized metals such as calcium. Note that the flux decreases sharply at wavelengths less than 3800 Å, this is called the *Balmer jump*. A similar *Paschen jump* appears at wavelengths that are  $3^2/2^2$  times longer, at around 8550 Å.

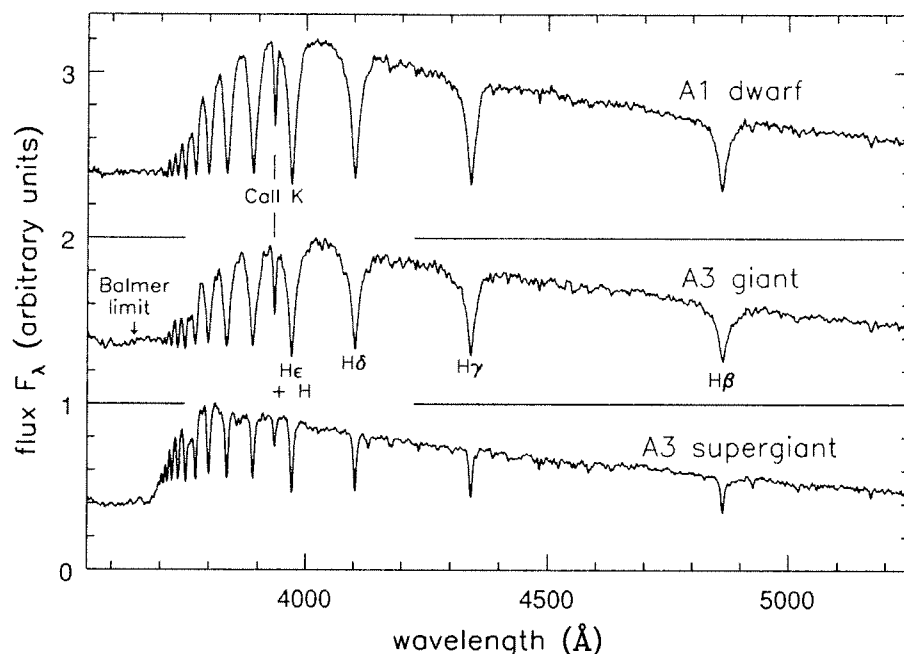
In F stars, the hydrogen lines are weaker than in A stars, and lines of neutral metals begin to appear. G stars, like the Sun, are cooler than about 6000 K. The most prominent absorption features are the ‘H and K’ lines of singly ionized calcium (CaII), and the G band of CH at 4300 Å. These were named in 1815 by Joseph Fraunhofer, who discovered the strong absorption lines in the Sun’s spectrum, and labelled them from A to K in order from red to blue. Lines of neutral metals, such as the pair of D lines of neutral sodium (NaI) at 5890 Å and 5896 Å, are stronger than in hotter stars.

In K stars, we see mainly lines of neutral metals and of molecules such as TiO, titanium oxide. At wavelengths below 4000 Å metal lines absorb much of the light, creating the *4000 Å break*. The spectrum of the M star, cooler than about 4000 K, shows deep absorption bands of TiO and of VO, vanadium oxide, as well as lines of neutral metals. This is not because M stars are rich in titanium, but because these molecules absorb red light very efficiently, and the atmosphere is cool enough that they do not break apart. L stars have surface temperatures below about 2500 K, and most of the titanium and vanadium in their atmospheres is condensed onto dust grains. Hence bands of TiO and VO are much weaker than in M stars; lines of neutral metals such as cesium appear, while the sodium D lines become very strong and broad. T stars are those with surfaces cooler than 1400 K; their spectra show strong lines of water and methane, like the atmospheres of giant planets.

We can measure masses for these dwarfs by observing them in binary systems, and comparing with evolutionary models. Such work indicates a mass  $\mathcal{M} \approx 0.15\mathcal{M}_\odot$  for a main-sequence M5 star, while  $\mathcal{M} \approx 0.08\mathcal{M}_\odot$  for a single measured L0–L1 binary. Counting the numbers of M, L, and T dwarfs in the solar neighborhood shows that objects below  $0.3\mathcal{M}_\odot$  contribute little to the total mass in the Milky Way’s thin disk. ‘Stars’ cooler than about L5 have too little mass to sustain hydrogen burning in their cores. They are not true stars, but *brown dwarfs*, cooling as they contract slowly under their own weight. Over its first 100 Myr or so, a given brown dwarf can cool from spectral class M to L, or even T; the temperature drops only slowly during its later life.

The spectrum of a galaxy is *composite*, including the light from a mixture of stars with different temperatures. The hotter stars give out most of the blue light, and the lines observed in the blue part of the spectrum of a galaxy such as the Milky Way are typically those of A, F, or G stars. O and B stars are rare and so do not contribute much of the visible light, unless a galaxy has had a recent burst of star formation. In the red part of the spectrum, we see lines from the cooler K stars, which produce most of the galaxy’s red light. Thus the blue part of the spectrum of a galaxy such as the Milky Way shows the Balmer lines of hydrogen in absorption, while TiO bands are present in the red region.

It is much easier to measure the strength of spectral lines relative to the flux at nearby wavelengths than to determine  $F_\lambda(\lambda)$  over a large range in wavelength. Absorption and scattering by dust in interstellar space, and by the Earth’s



**Fig. 1.2.** Spectra of an A1 dwarf, an A3 giant, and an A3 supergiant: the most luminous star has the narrowest spectral lines – G. Jacoby *et al.*, spectral library.

atmosphere, affects the blue light of stars more than the red; blue and red light also propagate differently through the telescope and the spectrograph. In practice, stellar temperatures are often estimated by comparing the observed depths of absorption lines in their spectra with the predictions of a *model stellar atmosphere*. This is a computer calculation of the way light propagates through a stellar atmosphere with a given temperature and composition; it is calibrated against stars for which  $F_\lambda$  has been measured carefully.

The lines in stellar spectra also give us information about the surface gravity. Figure 1.2 shows the spectra of three stars, all classified as A stars because the overall strength of their absorption lines is similar. But the Balmer lines of the A dwarf are broader than those in the giant and supergiant stars, because atoms in its photosphere are more closely crowded together: this is known as the *Stark effect*. If we use a model atmosphere to calculate the surface gravity of the star, and we also know its mass, we can then find its radius. For most stars, the surface gravity is within a factor of three of that in the Sun; these stars form the *main sequence* and are known as *dwarfs*, even though the hottest of them are very large and luminous.

All main-sequence stars are burning hydrogen into helium in their cores. For any particular spectral type, these stars have nearly the same mass and luminosity, because they have nearly identical structures: the hottest stars are the most massive, the most luminous, and the largest. Main-sequence stars have radii between  $0.1 R_\odot$

and about  $25R_{\odot}$ : very roughly,

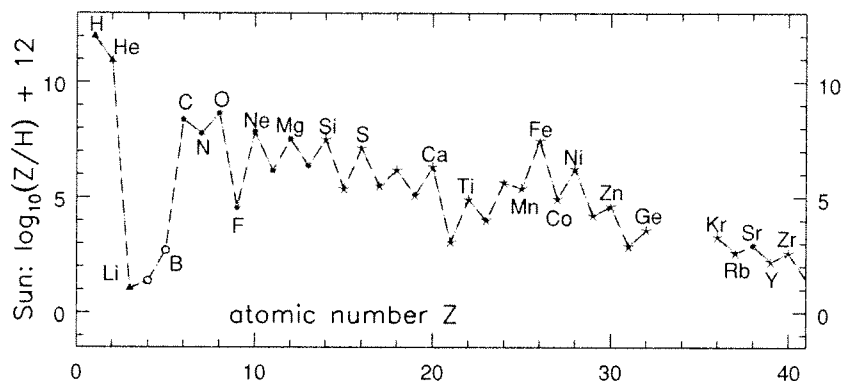
$$R \sim R_{\odot} \left( \frac{\mathcal{M}}{\mathcal{M}_{\odot}} \right)^{0.7} \quad \text{and} \quad L \sim L_{\odot} \left( \frac{\mathcal{M}}{\mathcal{M}_{\odot}} \right)^{\alpha}, \quad (1.6)$$

where  $\alpha \approx 5$  for  $\mathcal{M} \lesssim \mathcal{M}_{\odot}$ , and  $\alpha \approx 3.9$  for  $\mathcal{M}_{\odot} \lesssim \mathcal{M} \lesssim 10\mathcal{M}_{\odot}$ . For the most massive stars with  $\mathcal{M} \gtrsim 10\mathcal{M}_{\odot}$ ,  $L \sim 50L_{\odot}(\mathcal{M}/\mathcal{M}_{\odot})^{2.2}$ . *Giant* and *supergiant* stars have a lower surface gravity and are much more distended; the largest stars have radii exceeding  $1000R_{\odot}$ . Equation 1.3 tells us that they are much brighter than main-sequence stars of the same spectral type. Below, we will see that they represent later stages of a star's life. *White dwarfs* are not main-sequence stars, but have much higher surface gravity and smaller radii; a white dwarf is only about the size of the Earth, with  $R \approx 0.01R_{\odot}$ . If we define a star by its property of generating energy by nuclear fusion, then a white dwarf is no longer a star at all, but only the ashes or embers of a star's core; it has exhausted its nuclear fuel and is now slowly cooling into blackness. A *neutron star* is an even smaller stellar remnant, only about 20 km across, despite having a mass larger than the Sun's.

**Further reading:** for an undergraduate-level introduction to stars, see D. A. Ostlie and B. W. Carroll, 1996, *An Introduction to Modern Stellar Astrophysics* (Addison-Wesley, Reading, Massachusetts); and D. Prialnik, 2000, *An Introduction to the Theory of Stellar Structure and Evolution* (Cambridge University Press, Cambridge, UK).

The strength of a given spectral line depends on the temperature of the star in the layers where the line is formed, and also on the abundance of the various elements. By comparing the strengths of various lines with those calculated for a hot gas, Cecilia Payne-Gaposchkin showed in 1925 that the Sun and other stars are composed mainly of hydrogen. The surface layers of the Sun are about 72% hydrogen, 26% helium, and about 2% of all other elements, by mass. Astronomers refer collectively to the elements heavier than helium as *heavy elements* or *metals*, even though substances such as carbon, nitrogen, and oxygen would not normally be called metals.

There is a good reason to distinguish hydrogen and helium from the rest of the elements. These atoms were created in the aftermath of the Big Bang, less than half an hour after the Universe as we now know it came into existence; the neutrons and protons combined into a mix of about three-quarters hydrogen, one-quarter helium, and a trace of lithium. Since then, the stars have burned hydrogen to form helium, and then fused helium into heavier elements; see the next subsection. Figure 1.3 shows the abundances of the commonest elements in the Sun's photosphere. Even oxygen, the most plentiful of the heavy elements, is over 1000 times rarer than hydrogen. The 'metals' are found in almost, but not



**Fig. 1.3.** Logarithm of the number of atoms of each element found in the Sun, for every  $10^{12}$  hydrogen atoms. Hydrogen, helium, and lithium originated mainly in the Big Bang, the next two elements result from the breaking apart of larger atoms, and the remainder are ‘cooked’ in stars. Filled dots show elements produced mainly in quiescent burning; star symbols indicate those made largely during explosive burning in a supernova – M. Asplund *et al.*, astro-ph/0410214.

exactly, the same proportions in all stars. The small differences can tell us a lot about the history of the material that went into making a star; see Section 4.3.

The fraction by mass of the heavy elements is denoted  $Z$ : the Sun has  $Z_{\odot} \approx 0.02$ , while the most metal-poor stars in our Galaxy have less than 1/10 000 of this amount. If we want to specify the fraction of a particular element, such as oxygen, in a star, we often give its abundance relative to that in the Sun. We use a logarithmic scale:

$$[A/B] \equiv \log_{10} \left\{ \frac{(\text{number of A atoms/number of B atoms})_{\star}}{(\text{number of A atoms/number of B atoms})_{\odot}} \right\}, \quad (1.7)$$

where  $\star$  refers to the star and we again use  $\odot$  for the Sun. Thus, in a star with  $[\text{Fe}/\text{H}] = -2$ , iron is 1% as abundant as in the Sun. A warning:  $[\text{Fe}/\text{H}]$  is often used for a star’s average heavy-element abundance relative to the Sun; it does not always refer to measured iron content.

### I.1.3 The lives of the stars

Understanding how stars proceed through the different stages of their lives is one of the triumphs of astrophysics in the second half of the twentieth century. The discovery of nuclear-fusion processes during the 1940s and 1950s, coupled with the fast digital computers that became available during the 1960s and 1970s,



has given us a detailed picture of the evolution of a star from a protostellar gas cloud through to extinction as a white dwarf, or a fiery death in a supernova explosion.

We are confident that we understand most aspects of main-sequence stars fairly well. A long-standing discrepancy between predicted nuclear reactions in the Sun's core and the number of neutrinos detected on Earth was recently resolved in favor of the stellar modellers: neutrinos are produced in the expected numbers, but many had changed their type along the way to Earth. Our theories falter at the beginning of the process – we do not know how to predict when a gas cloud will form into stars, or what masses these will have – and toward its end, especially for massive stars with  $\mathcal{M} \gtrsim 8\mathcal{M}_{\odot}$ , and for stars closely bound in binary systems. This remaining ignorance means that we do not yet know what determines the rate at which galaxies form their stars; the quantity of elements heavier than helium that is produced by each type of star; and how those elements are returned to the interstellar gas, to be incorporated into future generations of stars.

The mass of a star almost entirely determines its structure and ultimate fate; chemical composition plays a smaller role. Stars begin their existence as clouds of gas that become dense enough to start contracting under the inward pull of their own gravity. Compression heats the gas, making its pressure rise to support the weight of the exterior layers. But the warm gas then radiates away energy, reducing the pressure, and allowing the cloud to shrink further. In this *protostellar* stage, the release of gravitational energy counterbalances that lost by radiation. As a protostar, the Sun would have been cooler than it now is, but several times more luminous. This phase is short: it lasted only 50 Myr for the Sun, which will burn for 10 Gyr on the main sequence. So protostars do not make a large contribution to a galaxy's light.

The temperature at the center rises throughout the protostellar stage; when it reaches about  $10^7$  K, the star is hot enough to 'burn' hydrogen into helium by thermonuclear fusion. When four atoms of hydrogen fuse into a single atom of helium, 0.7% of their mass is set free as energy, according to Einstein's formula  $\mathcal{E} = \mathcal{M}c^2$ . Nuclear reactions in the star's *core* now supply enough energy to maintain the pressure at the center, and contraction stops. The star is now quite stable: it has begun its *main-sequence* life. Table 1.1 gives the luminosity and effective temperature for stars of differing mass on the *zero-age main sequence*; these are calculated from models for the internal structure, assuming the same chemical composition as the Sun. Each solid track on Figure 1.4 shows how those quantities change over the star's lifetime. A plot like this is often called a *Hertzsprung–Russell* diagram, after Ejnar Hertzsprung and Henry Norris Russell, who realized around 1910 that, if the luminosity of stars is plotted against their spectral class (or color or temperature), most of the stars fall close to a diagonal line which is the main sequence. The temperature increases to the left on the horizontal axis to correspond to the ordering O B A F G K M of the spectral classes. As the star burns hydrogen to helium, the mean mass of its constituent

**Table 1.1** Stellar models with solar abundance, from Figure 1.4

<i>Mass</i> ( $\mathcal{M}_\odot$ )	$L_{\text{ZAMS}}$ ( $L_\odot$ )	$T_{\text{eff}}$ (K)	<i>Spectral</i> <i>type</i>	$\tau_{\text{MS}}$ (Myr)	$\tau_{\text{red}}$ (Myr)	$\int(L \, d\tau)_{\text{MS}}$ (Gyr $\times L_\odot$ )	$\int(L \, d\tau)_{\text{pMS}}$ (Gyr $\times L_\odot$ )
0.8	0.24	4860	K2	25 000		10	
1.0	0.69	5640	G5	9800	3200	10.8	24
1.25	2.1	6430		3900	1650	11.7	38
1.5	4.7	7110	F3	2700	900	16.2	13
2	16	9080	A2	1100	320	22.0	18
3	81	12 250	B7	350	86	38.5	19
5	550	17 180	B4	94	14	75.2	23
9	4100	25 150		26	1.7	169	40
15	20 000	31 050		12	1.1	360	67
25	79 000	37 930		6.4	0.64	768	145
40	240 000	43 650	O5	4.3	0.47	1500	112
60	530 000	48 190		3.4	0.43	2550	9
85	1 000 000	50 700		2.8		3900	
120	1 800 000	53 330		2.6		5200	

*Note:*  $L$  and  $T_{\text{eff}}$  are for the zero-age main sequence; spectral types are from Table 1.3;  $\tau_{\text{MS}}$  is main-sequence life;  $\tau_{\text{red}}$  is time spent later as a red star ( $T_{\text{eff}} \lesssim 6000$  K); integrals give energy output on the main sequence (MS), and in later stages (pMS).

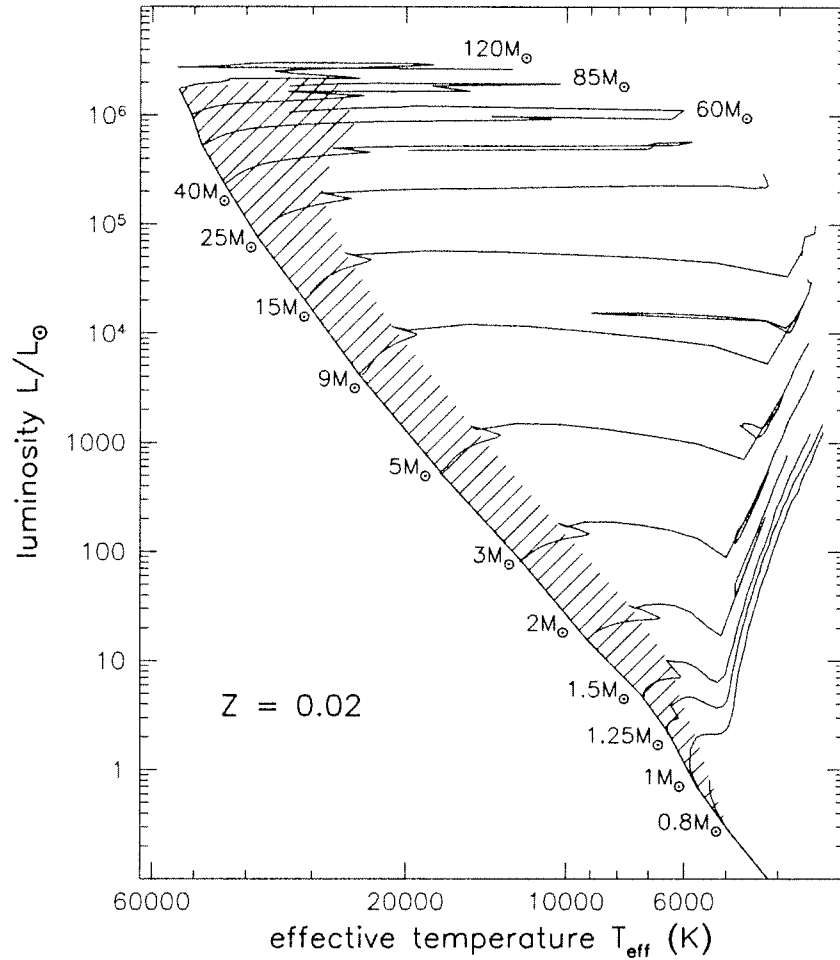
particles slowly increases, and the core must become hotter to support the denser star against collapse. Nuclear reactions go faster at the higher temperature, and the star becomes brighter. The Sun is now about 4.5 Gyr old, and its luminosity is almost 50% higher than when it first reached the main sequence.

**Problem 1.4** What mass of hydrogen must the Sun convert to helium each second in order to supply the luminosity that we observe? If it converted all of its initial hydrogen into helium, how long could it continue to burn at this rate? Since it can burn only the hydrogen in its core, and because it is gradually brightening, it will remain on the main sequence for only about 1/10 as long.

**Problem 1.5** Use Equation 1.3 and data from Table 1.1 to show that, when the Sun arrived on the main sequence, its radius was about  $0.87 R_\odot$ .

A star can continue on the main sequence until thermonuclear burning has consumed the hydrogen in its core, about 10% of the total. Table 1.1 lists the time  $\tau_{\text{MS}}$  that stars of each mass spend there; it is most of the star's life. So, at any given time, most of a galaxy's stars will be on the main sequence. For an average value  $\alpha \approx 3.5$  in Equation 1.6, we have

$$\tau_{\text{MS}} = \tau_{\text{MS},\odot} \frac{\mathcal{M}/L}{\mathcal{M}_\odot/L_\odot} \sim 10 \text{ Gyr} \left( \frac{\mathcal{M}}{\mathcal{M}_\odot} \right)^{-2.5} = 10 \text{ Gyr} \left( \frac{L}{L_\odot} \right)^{-5/7}. \quad (1.8)$$

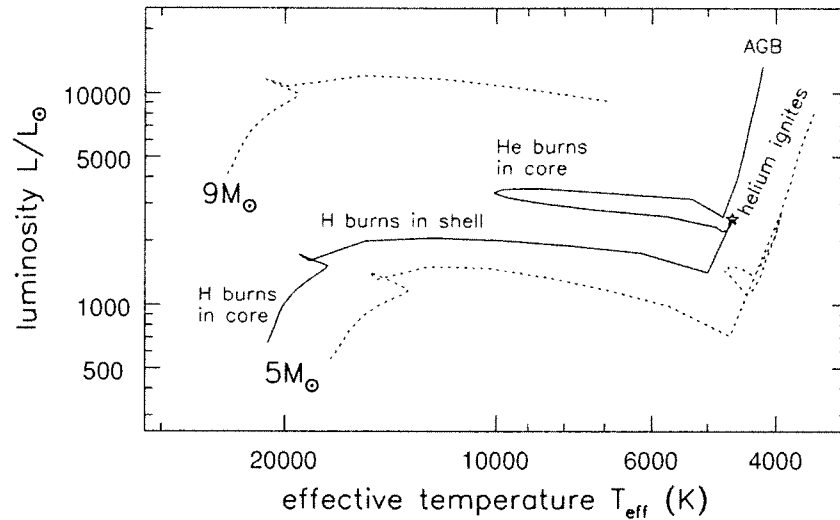


**Fig. 1.4.** Luminosity and effective temperature during the main-sequence and later lives of stars with solar composition: the hatched region shows where the star burns hydrogen in its core. Only the main-sequence track is shown for the  $0.8M_{\odot}$  star – Geneva Observatory tracks.

A better approximation is

$$\log(\tau_{\text{MS}}/10 \text{ Gyr}) = 1.015 - 3.49 \log(M/M_{\odot}) + 0.83[\log(M/M_{\odot})]^2. \quad (1.9)$$

The most massive stars will burn out long before the Sun. None of the O stars shining today were born when dinosaurs walked the Earth 100 million years ago, and all those we now observe will burn out before the Sun has made another circuit of the Milky Way. But we have not included any stars with  $M < 0.8M_{\odot}$  in Figure 1.4, because none has left the main sequence since the Big Bang,  $\sim 14$  Gyr



**Fig. 1.5.** Evolutionary tracks of a  $5M_{\odot}$  and a  $9M_{\odot}$  star with solar composition (dotted curves), and a metal-poor  $5M_{\odot}$  star with  $Z = 0.001 \approx Z_{\odot}/20$  (solid curve). The metal-poor star makes a ‘blue loop’ while burning helium in its core; it is always brighter and bluer than a star of the same mass with solar metallicity – Geneva Observatory tracks.

in the past. Most of the stellar mass of galaxies is locked into these dim long-lived stars.

Decreasing the fraction of heavy elements in a star makes it brighter and bluer; see Figure 1.5. The ‘metals’ are a source of *opacity*, blocking the escape of photons which carry energy outward from the core through the interior and the atmosphere. If the metal abundance is low, light moves to the surface more easily; as a result, a metal-poor star is more compact, meaning that it is denser. So its core must be hotter, and produce more energy. Consequently, the star uses up its nuclear fuel faster.

In regions of a star where photons carry its energy out toward the surface, collisions between atoms cannot mix the ‘ash’ of nuclear burning with fresh material further out. The star, which began as a homogeneous ball of gas, develops strata of differing chemical composition. Convection currents can stir up the star’s interior, mixing the layers. Our figures and table are computed for stars that do not spin rapidly on their axes. Fast rotation encourages mixing, and the fresh hydrogen brought into the star’s core extends its life on the main sequence.

At the end of its main-sequence life, the star leaves the hatched area in Figure 1.4. Its life beyond that point is complex and depends very much on the star’s mass. All stars below about  $0.6M_{\odot}$  stay on the main sequence for so long that none has yet left it in the history of the Universe. In *low-mass stars* with  $0.6M_{\odot} \lesssim M \lesssim 2M_{\odot}$ , the hydrogen-exhausted core gives out energy by shrinking; it becomes denser, while the star’s outer layers puff up to a hundred times their

former size. The star now radiates its energy over a larger area, so Equation 1.3 tells us that its surface temperature must fall; it becomes cool and red. This is the *subgiant* phase.

When the temperature just outside the core rises high enough, hydrogen starts to burn in a surrounding shell: the star becomes a *red giant*. Helium ‘ash’ is deposited onto the core, making it contract further and raising its temperature. The shell then burns hotter, so more energy is produced, and the star becomes gradually brighter. During this phase, the tracks of stars with  $M \lesssim 2M_{\odot}$  lie close together at the right of Figure 1.4, forming the red giant branch. Stars with  $M \lesssim 1.5M_{\odot}$  give out most of their energy as red giants and in later stages; see Table 1.1. By contrast with main-sequence stars, the luminosity and color of a red giant depend very little on its mass; so the giant branches in stellar systems of different ages can be very similar. Just as on the main sequence, stars with low metallicity are somewhat bluer and brighter.

As it contracts, the core of a red giant becomes dense enough that the electrons of different atoms interact strongly with each other. The core becomes *degenerate*; it starts to behave like a solid or a liquid, rather than a gas. When the temperature at its core has increased to about  $10^8$  K, helium ignites, burning to carbon; this releases energy that heats the core. In a gas, expansion would dampen the rate of nuclear reactions to produce a steady flow of energy. But the degenerate core cannot expand; instead, like a liquid or solid, its density hardly changes, so burning is explosive, as in an uncontrolled nuclear reactor on Earth. This is the *helium flash*, which occurs at the very tip of the red giant branch in Figure 1.4. In about 100 s, the core of the star heats up enough to turn back into a normal gas, which then expands.

On the red giant branch, the star’s luminosity is set by the mass of its helium core. When the helium flash occurs, the core mass is almost the same for all stars below  $\sim 2M_{\odot}$ ; so these stars should reach the same luminosity at the tip of the red giant branch. In any stellar population more than 2–3 Gyr old, stars above  $2M_{\odot}$  have already completed their lives; if the metal abundance is below  $\sim 0.5Z_{\odot}$ , the red giants have almost the same color. So the apparent brightness at the tip of the red giant branch can be used to find the distance of a nearby galaxy.

Helium is now steadily burning in the core, and hydrogen in a surrounding shell. In Figure 1.4, we see that stars of  $M_{\odot}$  to  $2M_{\odot}$  stay cool and red during this phase; they are *red clump* stars. In Figure 2.2, showing the luminosity and color of stars close to the Sun, we see a concentration of stars in the red clump. Blue *horizontal branch* stars are in the same stage of burning. In these, little material remains in the star’s outer envelope, so the outer gas is relatively transparent to radiation escaping from the hot core. Stars that are less massive or poorer in heavy elements than the red clump will become horizontal branch stars.

Helium burning provides less energy than hydrogen burning. We see from Table 1.1 that this phase lasts no more than 30% as long as the star’s main-sequence life. Once the core has used up its helium, it must again contract, and

the outer envelope again swells. The star moves onto the *asymptotic giant branch* (AGB); it now burns both helium and hydrogen in shells, and it is more luminous and cooler than it was as a red giant. This is as far as we can follow its evolution in Figure 1.4.

On the AGB, both of the shells undergo pulses of very rapid burning, during which the loosely held gas of the outer layers is lost as a *stellar superwind*. Eventually the hot naked core is exposed, as a *white dwarf*: its ultraviolet radiation ionizes the ejected gas, which is briefly seen as a *planetary nebula*. White dwarfs near the Sun have masses around  $0.6M_{\odot}$ , meaning that at least half of the star's original material has been lost. The white dwarf core can do no further burning, and it gradually cools.

Stars of *intermediate mass*, from  $2M_{\odot}$  up to  $6M_{\odot}$  or  $8M_{\odot}$ , follow much the same history, up to the point when helium ignites in the core. Because their central density is lower at a given temperature, the helium core does not become degenerate before it begins to burn. These stars also become red, but Figure 1.4 shows that they are brighter than red giants; their tracks lie above the place where those of the lower-mass stars come together. Once helium burning is under way, the stars become bluer; some of them become *Cepheid variables*, F- and G-type supergiant stars which pulsate with periods between one and fifty days. Cepheids are very useful to astronomers, because the pulsation cycle betrays the star's luminosity: the most massive stars, which are also the most luminous, have the longest periods. So once we have measured the period and apparent brightness, we can use Equation 1.1 to find the star's distance. Cepheids are bright enough to be seen far beyond the Milky Way. In the 1920s, astronomers used them to show that other galaxies existed outside our own.

Once the core has used up its helium, these stars become red again; they are asymptotic giant branch stars, with both hydrogen and helium burning in shells. Rapid pulses of burning dredge gas up from the deep interior, bringing to the surface newly formed atoms of elements such as carbon, and heavier atoms which have been further 'cooked' in the star by the *s-process*: the slow capture of neutrons. For example, the atmospheres of some AGB stars show traces of the short-lived radioactive element technetium. The stellar superwind pushes polluted surface gas out into the interstellar environment; these AGB stars are a major source of the elements carbon and nitrogen in the Galaxy.

An intermediate-mass star makes a spectacular planetary nebula, as its outer layers are shed and subsequently ionized by the hot central core. The core then cools to become a white dwarf. Stars at the lower end of this mass range leave a core which is mainly carbon and oxygen; remnants of slightly more massive stars are a mix of oxygen, neon, and magnesium. We know that white dwarfs cannot have masses above  $1.4M_{\odot}$ ; so these stars put most of their material back into the interstellar gas.

In *massive stars*, with  $M \gtrsim 8M_{\odot}$ , the carbon, oxygen, and other elements left as the ashes of helium burning will ignite in their turn. The star Betelgeuse

is now a red supergiant burning helium in its core. It probably began its main-sequence life 10–20 Myr ago, with a mass between  $12M_{\odot}$  and  $17M_{\odot}$ . It will start to burn heavier elements, and finally explode as a supernova, within another 2 Myr. After their time on the main sequence, massive stars like Betelgeuse spend most of their time as blue or yellow supergiants; Deneb, the brightest star in the constellation Cygnus, is a yellow supergiant. Helium starts to burn in the core of a  $25M_{\odot}$  star while it is a blue supergiant, only slightly cooler than it was on the main sequence. Once the core's helium is exhausted, this star becomes a red supergiant; but mass loss can then turn it once again into a blue supergiant before the final conflagration.

The later lives of stars with  $M \gtrsim 40M_{\odot}$  are still uncertain, because they depend on how much mass has been lost through strong *stellar winds*, and on ill-understood details of the earlier convective mixing. A star of about  $50M_{\odot}$  may lose mass so rapidly that it never becomes a red supergiant, but is stripped to its nuclear-burning core and is seen as a blue *Wolf-Rayet* star. These are very hot stars, with characteristic strong emission lines of helium, carbon, and nitrogen coming from a fast stellar wind; the wind is very poor in hydrogen, since the star's outer layers were blown off long before. Wolf-Rayet stars live less than 10 Myr, so they are seen only in regions where stars have recently formed.

Once helium burning has finished in the core, a massive star's life is very nearly over. The carbon core quietly burns to neon, magnesium, and heavier elements. But this process is rapid, giving out little energy; most of that energy is carried off by *neutrinos*, weakly interacting particles which easily escape through the star's outer layers. A star that started on its main-sequence life with  $10M_{\odot} \lesssim M \lesssim 40M_{\odot}$  will burn its core all the way to iron. Such a core has no further source of energy. Iron is the most tightly bound of all nuclei, and it would *require* energy to combine its nuclei into yet heavier elements. The core collapses, and its neutrons are squeezed so tightly that they become degenerate. The outer layers of the star, falling in at a tenth of the speed of light, bounce off this suddenly rigid core, and are ejected in a blazing *Type II supernova*. Supernova 1987A which exploded in the Large Magellanic Cloud was of this type, which is distinguished by strong lines of hydrogen in its spectrum. The core of the star, incorporating the heavier elements such as iron, is either left as a *neutron star* or implodes as a *black hole*. The gas that escapes is rich in oxygen, magnesium, and other elements of intermediate atomic mass.

A star with an initial mass between  $8M_{\odot}$  and  $10M_{\odot}$  also ends its life as a Type II supernova, but by a slightly different process; the core probably collapses before it has burned to iron. After the explosion, a neutron star may remain, or the star may blow itself apart completely, like the Type Ia supernovae described below. A Wolf-Rayet star also becomes a supernova. Because its hydrogen has been lost, hydrogen lines are missing from the spectrum, and it is classified as Type Ic. These supernovae may be responsible for the energetic  $\gamma$ -ray bursts that

we discuss in Section 9.2. We shall see in Section 2.1 that massive stars are only a tiny fraction of the total; but they are a galaxy's main producers of oxygen and heavier elements. Detailed study of their later lives can tell us how much of each element is returned to the interstellar gas by stellar winds or supernova explosions, and how much will be locked within a remnant neutron star or black hole. In Section 4.3 we will discuss what the abundances of the various elements may tell us about the history of our Galaxy and others.

**Further reading:** see the books by Ostlie and Carroll, and by Prialnik. For stellar life beyond the main sequence, see the graduate-level treatment of D. Arnett, 1996, *Supernovae and Nucleosynthesis* (Princeton University Press, Princeton, New Jersey).

#### 1.1.4 Binary stars

Most stars are not found in isolation; they are in *binary or multiple star systems*. Binary stars can easily appear to be single objects unless careful measurements are made, and astronomers often say that 'three out of every two stars are in a binary'. Most binaries are widely separated, and the two stars evolve much like single stars. These systems cause us difficulty only because usually we cannot see the two stars as separate objects, even in nearby galaxies. When we observe them, we get a blend of two stars while thinking that we have only one.

In a close binary system, one star may remove matter from the other. It is especially easy to 'steal' gas from a red giant or an AGB star, since the star's gravity does not hold on strongly to the puffed-up outer layers. Then we can have some dramatic effects. For example, if one of the two stars becomes a white dwarf, hydrogen-rich gas from the companion can pour onto its surface, building up until it becomes dense enough to burn explosively to helium, in a sudden flash which we see as a *classical nova*. If the more compact star has become a neutron star or a black hole, gas falls onto its surface with such force that it is heated to X-ray-emitting temperatures.

A white dwarf in a binary can also explode as a *Type Ia supernova*. Such supernovae lack hydrogen lines in their spectra; they result from the explosive burning of carbon and oxygen. If the white dwarf takes enough matter from its binary companion, it can be pushed above the *Chandrasekhar limit* at about  $1.4M_{\odot}$ . No white dwarf can be heavier than this; if it gains more mass, it is forced to collapse, like the iron core in the most massive stars. But unlike that core, the white dwarf still has nuclear fuel: its carbon and oxygen burn to heavier elements, releasing energy which blows it apart. There is no remnant: the iron and other elements are scattered to interstellar space. Much of the iron we now find in the Earth and in the Sun has been produced in these supernovae. Even though close binary stars are relatively rare, they make a significant difference to the life of their host galaxy.