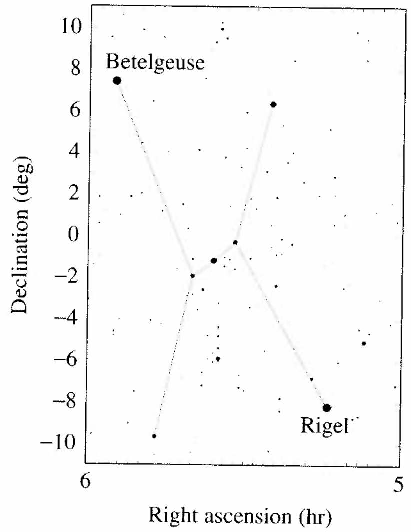

Figure 3.7 The constellation of Orion.

determining some aspects of the behavior of extremely luminous objects such as early main-sequence stars, red supergiants, or accreting compact stars. It may also have a significant effect on the small particles of dust found throughout the interstellar medium.

3.4 Blackbody Radiation

Anyone who has looked at the constellation of Orion on a clear winter night has noticed the strikingly different colors of red Betelgeuse (in Orion's northeast shoulder) and blue-white Rigel (in the southwest leg); see Fig. 3.7. These colors betray the difference in the surface temperatures of the two stars. Betelgeuse has a surface temperature of about 3400 K, significantly cooler than the 10,100 K surface of Rigel.[13]

The connection between the color of light emitted by a hot object and its temperature was first noticed in 1792 by the English maker of fine porcelain, Thomas Wedgewood. All of his ovens became red-hot at the same temperature, independent of their size, shape, and construction. Subsequent investigations by many physicists revealed that any object with a temperature above absolute zero emits light of all wavelengths with varying degrees of efficiency; an *ideal emitter* is an object that absorbs *all* of the light energy incident upon it,

[13]Both of these stars are pulsating variables (Chapter 14), so the values quoted are *average* temperatures.

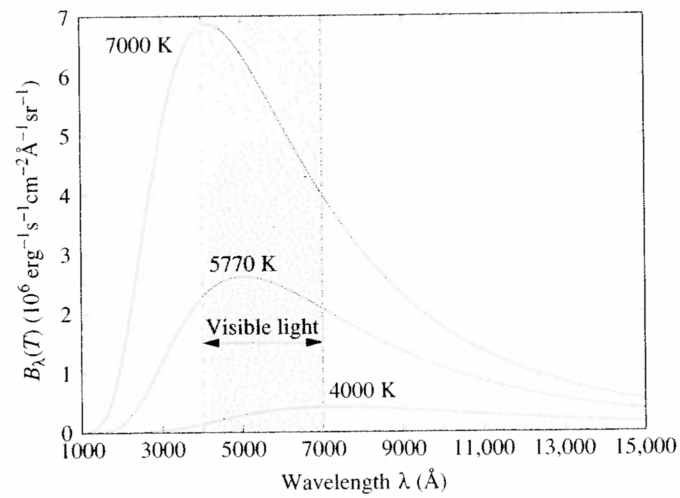

Figure 3.8 Blackbody spectrum [Planck function $B_\lambda(T)$].

and reradiates this energy with the characteristic spectrum shown in Fig. 3.8. Because an ideal emitter reflects no light, it is known as a **blackbody**, and the radiation it emits is called **blackbody radiation**. Stars and planets are blackbodies, at least to a rough first approximation.

Figure 3.8 shows that a blackbody of temperature T emits a **continuous spectrum** with some energy at all wavelengths and that this blackbody spectrum peaks at a wavelength λ_{max}, which becomes shorter with increasing temperature. The relation between λ_{max} and T is known as **Wien's displacement law**:[14]

$$\lambda_{\text{max}} T = 0.290 \text{ cm K}. \tag{3.15}$$

Example 3.4 Betelgeuse has a surface temperature of 3400 K. If we treat Betelgeuse as a blackbody, Wien's displacement law shows that its continuous spectrum peaks at a wavelength of

$$\lambda_{\text{max}} = \frac{0.290 \text{ cm K}}{3400 \text{ K}} = 8.53 \times 10^{-5} \text{ cm} = 8530 \text{ Å},$$

which is in the infrared region of the electromagnetic spectrum. Rigel, with a surface temperature of 10,100 K, has a continuous spectrum that peaks at a

[14] In 1911, the German physicist Wilhelm Wien received the Nobel Prize for his theoretical contributions to understanding the blackbody spectrum.

wavelength of

$$\lambda_{\rm max} = \frac{0.290 \text{ cm K}}{10,100 \text{ K}} = 2.87 \times 10^{-5} \text{ cm} = 2870 \text{ Å},$$

in the ultraviolet region.

Figure 3.8 also shows that as the temperature of a blackbody increases, it emits more energy per second at *all* wavelengths. Experiments performed by the Austrian physicist Josef Stefan in 1879 showed that the luminosity, L, of a blackbody of area A and temperature T (in kelvin) is given by

$$L = A\sigma T^4. \tag{3.16}$$

Five years later another Austrian physicist, Ludwig Boltzmann, derived this equation, now called the **Stefan–Boltzmann equation**, using the laws of thermodynamics and Maxwell's formula for radiation pressure. The Stefan–Boltzmann constant, σ, has the value

$$\sigma = 5.670 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}.$$

For a spherical star of radius R and surface area $A = 4\pi R^2$, the Stefan–Boltzmann equation takes the form

$$L = 4\pi R^2 \sigma T_e^4. \tag{3.17}$$

Since stars are not perfect blackbodies, we use this equation to *define* the **effective temperature** T_e of a star's surface. Combining this with the inverse square law, Eq. (3.2), shows that at the surface of the star ($r = R$), the *surface flux* is

$$F_{\rm surf} = \sigma T_e^4. \tag{3.18}$$

Example 3.5 The luminosity of the Sun is $L_\odot = 3.826 \times 10^{33}$ erg s^{-1} and its radius is $R_\odot = 6.960 \times 10^{10}$ cm. The effective temperature of the Sun's surface is then

$$T_\odot = \left(\frac{L_\odot}{4\pi R_\odot^2 \sigma}\right)^{\frac{1}{4}} = 5770 \text{ K}.$$

The radiant flux at the solar surface is

$$F_{\rm surf} = \sigma T_\odot^4 = 6.285 \times 10^{10} \text{ erg s}^{-1} \text{ cm}^{-2}.$$

According to Wien's displacement law, the Sun's continuous spectrum peaks at a wavelength of

$$\lambda_{\text{max}} = \frac{0.290 \text{ cm K}}{5770 \text{ K}} = 5.03 \times 10^{-5} \text{ cm} = 5030 \text{ Å}.$$

This wavelength falls in the *green* region (4910 Å $< \lambda <$ 5750 Å) of the spectrum of visible light. However, the Sun emits a continuum of wavelengths both shorter and longer than λ_{max}, and the human eye perceives the Sun's color as yellow. Because the Sun emits most of its energy at visible wavelengths (see Fig. 3.8), and because Earth's atmosphere is transparent at these wavelengths, the evolutionary process of natural selection has produced a human eye sensitive to this wavelength region of the electromagnetic spectrum.

Rounding off λ_{max} and $T_\odot$ to the more easily remembered values of 5000 Å and 5800 K, respectively, permits Wien's displacement law to be written in the convenient form

$$\lambda_{\text{max}} T = (5000 \text{ Å})(5800 \text{ K}). \tag{3.19}$$

This section draws to a close at the end of the nineteenth century. The physicists and astronomers of the time believed that all of the principles that govern the physical world had finally been discovered. Their scientific world view, the *Newtonian paradigm*, was the culmination of the heroic, golden age of classical physics that had flourished for over three hundred years. The construction of this paradigm began with the brilliant observations of Galileo and the subtle insights of Newton. Its architecture was framed by Newton's laws, supported by the twin pillars of the conservation of energy and momentum and illuminated by Maxwell's electromagnetic waves. Its legacy was a deterministic description of a universe that ran like clockwork, with wheels turning inside of wheels, all of its gears perfectly meshed. Physics was in danger of becoming a victim of its own success. There were no challenges remaining. All of the great discoveries apparently had been made, and the only task remaining for the men and women of science at the turn of the century was the filling in of details.

However, as the twentieth century opened, it became increasingly apparent that a crisis was brewing. Physicists were frustrated by their inability to answer some of the simplest questions concerning light. What is the medium through which light waves travel the vast distances between the stars, and what is Earth's speed through this medium? What determines the continuous spectrum of blackbody radiation and the characteristic, discrete colors of tubes filled with hot glowing gases? Astronomers were tantalized by hints of a treasure of knowledge just beyond their grasp.

It took a physicist of the stature of Albert Einstein to topple the Newtonian paradigm and bring about two revolutions in physics. One transformed our ideas about space and time, and the other changed our basic concepts of matter and energy. The rigid clockwork universe of the golden era was found to be an illusion and was replaced by a random universe governed by the laws of probability and statistics. The following four lines aptly summarize the situation. The first two lines were written by the English poet Alexander Pope, a contemporary of Newton; the last two, by J. C. Squire, are of a more recent vintage.

Nature and Nature's laws lay hid in night:
God said, *Let Newton be!* and all was light.

It did not last: the Devil howling "Ho!
Let Einstein be!" restored the status quo.

3.5 The Quantization of Energy

By late 1900 the German physicist Max Planck (1858–1947) had discovered an empirical formula that fit the blackbody spectra shown in Fig. 3.8:

$$B_\lambda(T) = \frac{a/\lambda^5}{e^{b/\lambda T} - 1},$$

where a and b are constants. In spherical coordinates, the amount of energy per unit time of radiation having wavelength between λ and $\lambda + d\lambda$ emitted by a blackbody of temperature T and surface area dA into a solid angle $d\Omega \equiv \sin\theta\, d\theta\, d\phi$ is given by

$$B_\lambda(T)\, d\lambda\, dA\, \cos\theta\, d\Omega = B_\lambda(T)\, d\lambda\, dA\, \cos\theta\, \sin\theta\, d\theta\, d\phi;$$

see Fig. 3.9.[15] The units of B_λ are therefore erg s^{-1} cm^{-3} sr^{-1}. Unfortunately, these units can be misleading. The reader should note that "erg cm^{-3}" indicates an energy per unit area per unit wavelength interval, erg cm^{-2} cm^{-1}, *not* an energy per unit volume. To help avoid confusion, the units of the wavelength interval $d\lambda$ are sometimes expressed in angstroms rather than centimeters, so the units of the Planck function become erg s^{-1} cm^{-2} $Å^{-1}$ sr^{-1}, as in Fig. 3.8.[16]

[15] Note that $dA\cos\theta$ is the area dA projected onto a plane perpendicular to the direction in which the radiation is traveling. The concept of a solid angle will be fully described in Section 6.1.

[16] The value of the Planck function thus depends on the units of the wavelength interval. The conversion of $d\lambda$ from centimeters to angstroms means that the values of B_λ obtained by evaluating Eq. (3.20) must be divided by 10^8.