AST 358 / Homework 5 Solution Set

1 From the tight correlation between BH mass M_{BH} and bulge velocity dispersion σ_B, shown in class (see slide 16 of posted powerpoint lec17-19 ; Fig 1) We can infer

$$M_{BH} \sim 5 \times 10^8 \, M_\odot \quad \text{for} \quad \sigma_B \sim 300 \text{ km/s}$$

Eddington luminosity of BH

$$L_{edd} = 1.3 \times 10^{38} \text{ erg s}^{-1} \left(\frac{M_{BH}}{M_\odot}\right)$$

$$= 6.5 \times 10^{46} \text{ erg s}^{-1}$$

2 Circular orbital speed V_c at radius R is related to the mass $M(R)$ enclosed inside radius R by

• $$\frac{V_c^2}{R} = \frac{G M(R)}{R^2}$$

$$M(R) = \frac{V_c^2 R}{G} = 222 \left(\frac{V_c}{\text{km s}^{-1}}\right)^2 \left(\frac{R}{pc}\right) M_\odot$$

$$= 222 \times (10\,000)^2 \times 2$$

$$= 4.4 \times 10^{10} \, M_\odot$$

• If this mass M was a compact stellar cluster, then the average no N of stars of mass $1 M_\odot$ is

$$N \sim 4.4 \times 10^{10}$$

The no density of stars n is

$$n = \frac{N}{\frac{4}{3}\pi R^3} = 4.4 \times 10^{10} \Big/ \frac{4}{3}\pi \times (2 \times 3 \times 10^{16} \text{ m})^3$$

$$n \sim 5 \times 10^{-41} \ m^{-3}$$ (1)

Stellar collision timescale is given by

$$t_{coll} = \frac{1}{n \sigma A} = \frac{\text{mean free path } \lambda}{\sigma}$$

Where σ = typical speed of star

A = cross section of star

$= \pi R_{\odot}^2$

$= \pi \, (7 \times 10^8 \, m)^2 = 1.5 \times 10^{18} \ m^2$ (2)

Take $\sigma \approx V_c \sim 10^4$ km/s $= 10^7$ m/s

$$t_{coll} = \frac{1}{n \sigma A} = 1.3 \times 10^{15} s \sim \left(\frac{1.3 \times 10^{15}}{3 \times 10^7} \right) \text{yrs}$$

$$\sim 4 \times 10^7 \ yr$$

So $t_{coll} \ll$ Hubble time of $\sim 10^{10}$ yrs

This means that a stellar cluster of mass $M \sim 4 \times 10^{10} M_{\odot}$ and radius $R \sim 2$ pc would be dynamically unstable. Stellar collisions would cause such a system to be disrupted: the central part would undergo core collapse and the outer stars would be flung out and escape.

Thus, the object is not a massive stellar cluster and is likely to be a black hole.

Q3

a) $$1+z = \frac{\lambda_{obs}}{\lambda_{rest}} = \frac{3281.4}{656.3} = 5$$

$$z \sim 4$$

b) Lookback time T_b = Age today − Age at redshift $z \sim 4$

$$= 13.7 - 1.5 \quad \text{from Fig 1}$$

$$= 12.2 \text{ Gyr}$$

$$T_b \text{ as \% of present age} = \frac{12.2}{13.7} \times 100 = 89\ \%$$

c) $$I = I_0 \, e^{-R/R_d}$$

At point where $I = \frac{I_0}{10}$, $\frac{R}{R_d} = \ln(10)$

$$\rightarrow \quad R = 2.3 \; R_d$$

$$= 2.3 \times 0.5''$$

$$= 1.15''$$

From Fig 2, angular diameter distance D_A at $z \sim 4$ is 1400 Mpc

From class: $$(R \text{ in kpc}) = \frac{(R \text{ in } '') \times (D_A \text{ in Mpc})}{206.3}$$

$$= \frac{1.15 \times 1400}{206.3} = 7.8 \text{ kpc}$$

(d) Observed surface brightness = Intrinsic surface brightness $\cdot (1+z)^{-4}$

$$I_{obs} = I_{int}(1+z)^{-4}$$

$$m_{obs} - m_{int} = -2.5 \lg\left(\frac{I_{obs}}{I_{int}}\right)$$

$$= -2.5 \lg (1+z)^{-4}$$

$$= -2.5 \times (-4) \lg(5)$$

$$= 7$$

Observed surf. brightness = 25 mag arcsec^{-2} + 7 mag arcsec^{-2}

= 32 mag arcsec^{-2}