## ASTRO 358: HOMEWORK 5

## Assigned on Th Apr 19, 2012. Due by noon on Th Apr 26, 2012

Instructions: In order to get full credit, you must show the method that you used to derive the answer. The number of points for each question is indicated in brackets, and the total score is 100 points.

1. Consider a galaxy whose bulge has a velocity dispersion of $300 \mathrm{~km} \mathrm{~s}^{-1}$. Estimate the Eddington luminosity for mass accretion onto the central black hole, stating the assumptions you make. [20 pts]
2. Ionized gas in a galaxy is observed to be moving at a speed of $10,000 \mathrm{~km} \mathrm{~s}^{-1}$ at a radius of 2 pc. Assuming that the gas is on a circular orbit, estimate the mass enclosed inside this radius. Why is this massive central object considered to be a black hole rather than a massive compact stellar cluster? $[\mathbf{2 0}+\mathbf{1 0}=\mathbf{3 0} \mathbf{~ p t s}]$
3. For this question use Figures 1 and 2, which show the age of the Universe and the angular diameter distance $D_{\mathrm{A}}$ as a function of cosmological redshift $z$, for a cosmology with $\Omega_{\mathrm{M}}=0.3$, $\Omega_{\Lambda}=0.7, \Omega_{\mathrm{k}}=0$, and $H_{0}=70 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{Mpc}^{-1}$.
(a) The $\mathrm{H} \alpha$ line has a rest wavelength of 656.285 nm . The spectrum of a distant galaxy $G 1$ shows this line redshifted to a wavelength $\lambda$ of 3281.425 nm . Calculate the redshift $z$ of the galaxy $G 1$. [10 pts]
(b) Assume that the above redshift is due purely due to the cosmological expansion of the Universe. What is the lookback time corresponding to this redshift? Expression your answer as a fraction of the present age of the Universe. [10 pts]
(c) The galaxy $G 1$ has a disk that shows an exponential radial surface brightness profile with a scale length $R_{\mathrm{s}} \sim 0.5^{\prime \prime}$. Estimate the radius where the surface brightness is $1 / 10$ of its peak value, expressing your answer in kpc. [10 pts]
(d) Assume that the surface brightness of a source dims at the rate of $(1+z)^{-4}$. If a galaxy with an intrinsic surface brightness of 25 magnitude per $\operatorname{arcsec}^{2}$ were observed at the same redshift as galaxy $G 1$, what would be its observed surface brightness in magnitude per $\operatorname{arcsec}^{2}$. [20 $\mathbf{p t s}$ ]


Fig. 1 - The age of the Universe is plotted versus redshift $z$ for a cosmological model with $\Omega_{\mathrm{M}}$ $=0.3, \Omega_{\Lambda}=0.7, \Omega_{\mathrm{k}}=0$, and $H_{0}=70 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{Mpc}^{-1}$. Assume an age of 13.7 Gyr at $z \sim 0$.


Fig. 2 - The angular diameter distance $D_{\mathrm{A}}$ is plotted versus redshift $z$ for a cosmological model with $\Omega_{\mathrm{M}}=0.3, \Omega_{\Lambda}=0.7, \Omega_{\mathrm{k}}=0$, and $H_{0}=70 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{Mpc}^{-1}$.

## Values of physical constants

$1 M_{\odot}=2 \times 10^{30} \mathrm{~kg}$
Proton mass $=1.67 \times 10^{-27} \mathrm{~kg}$
Electron mass $=9.1 \times 10^{-31} \mathrm{~kg}$
1 parsec ( pc ) $=3 \times 10^{16} \mathrm{~m}$.
$G=$ Gravitational constant $=6.7 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} \mathrm{~kg}^{-2}=4.5 \times 10^{-3}\left(\mathrm{~km} \mathrm{~s}^{-1}\right)^{2} \mathrm{pc} M_{\odot}{ }^{-1}$
Wien's constant $W=2.9 \times 10^{-3} \mathrm{~m} \mathrm{~K}$
$c=$ Speed of light $=3 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$
Planck's constant $h=6.6 \times 10^{-34} \mathrm{~J} \mathrm{~s}$;
Hubble's constant $H_{0}=75 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{Mpc}^{-1}$
$\sigma=$ Stefan-Boltzmann constant $=5.7 \times 10^{-8} \mathrm{~J}$
$k_{\mathrm{B}}=$ Boltzmann constant $=1.38 \times 10^{-23} \mathrm{~J} \mathrm{~K}^{-1}$
1 Joule $=10^{7} \mathrm{erg}$

## END OF ASSIGNMENT

