## ASTRO 358: HOMEWORK 3

## Assigned on Tu Mar 27, 2012. Due by noon on Tu Apr 3, 2012 in class

Instructions: In order to get full credit, you must show the method that you used to derive the answer. The number of points for each question is indicated in brackets, and the total score is 50 points.

1. For this question use the rotation curve in Figure 1, shown on the next page.
(a) Using the plotted axes below Fig. 1, sketch the epicyclic frequency $\kappa$ as a function of radius $r$ over the solid body and flat part of the rotation curve, and label the peak value of $\kappa$ in each region, in units of $\mathrm{km} \mathrm{s}^{-1} \mathrm{pc}^{-1}$. Leave the transition region blank. [ $\mathbf{1 5} \mathbf{~ p t s ] ~}$
(b) Assume that within the central 350 pc radius of the galaxy there is a purely gaseous disk, which has a gas surface density $\Sigma_{\text {gas }}$ of $2000 M_{\odot} \mathrm{pc}^{-2}$, a gas velocity dispersion $\sigma$ of $30 \mathrm{~km} \mathrm{~s}^{-1}$, and the epicyclic frequency $\kappa$ that you calculated in part (a). Determine whether the gas in this region is unstable to gravitational instabilities by estimating the Toomre $Q$ parameter. In this problem, you can assume that the disk is axisymmetric and infinitely thin. [20 pts]
2. An $\mathrm{H} \alpha$ flux of $1.6 \times 10^{-13} \mathrm{~J} \mathrm{~s}^{-1} \mathrm{~m}^{-2}$ is received from a galaxy in the Virgo cluster at a distance of 20 Mpc . What is the star formation rate of this galaxy in $M_{\odot} \mathrm{yr}^{-1}$ ? [ $\mathbf{1 5} \mathbf{~ p t s ]}$

## Values of physical constants

Wien's constant $W=2.9 \times 10^{-3} \mathrm{~m} \mathrm{~K}$
$\sigma=$ Stefan-Boltzmann constant $=5.7 \times 10^{-8} \mathrm{~J} \mathrm{~s}^{-1} \mathrm{~m}^{-2} \mathrm{~K}^{-4}$
$G=$ Gravitational constant $=6.7 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} \mathrm{~kg}^{-2}=4.5 \times 10^{-3}\left(\mathrm{~km} \mathrm{~s}^{-1}\right)^{2} \mathrm{pc} M_{\odot}{ }^{-1}$
$c=$ Speed of light $=3 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$
$k_{\mathrm{B}}=$ Boltzmann constant $=1.38 \times 10^{-23} \mathrm{~J} \mathrm{~K}^{-1}$
Hubble's constant $H_{0}=70 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{Mpc}^{-1}$
Planck's constant $h=6.6 \times 10^{-34} \mathrm{~J} \mathrm{~s}$
Proton mass $=1.67 \times 10^{-27} \mathrm{~kg}$
$1 M_{\odot}=2 \times 10^{30} \mathrm{~kg}$
1 parsec $(\mathrm{pc})=3 \times 10^{16} \mathrm{~m}$
1 Joule $=10^{7} \mathrm{erg}$


Fig. 1 - The circular speed $V_{\mathrm{c}}(R)$ is plotted as a function of galactocentric radius $R$ in a disk galaxy. The rotation curve shows 3 regions: a solid body part where $V_{\mathrm{c}}$ is proportional to radius out to $R=350 \mathrm{pc}$, a flat part at $R \geq 500 \mathrm{pc}$ where $V_{\mathrm{c}}$ is independent of $R$, and a transition region in between.

