AST 358 Hwk 2 | Q 1

(a)

$$\frac{G M_R m}{R^2} = \frac{m v^2}{R}$$ for a star of mass m, speed V

$$M_R = \frac{V^2 R}{G}$$

$$\left(\frac{M_R}{M_\odot}\right) = 2.2 \times 10^5 \left(\frac{V}{km s^{-1}}\right)^2 \left(\frac{R}{kpc}\right) \quad (1)$$

For $R = 10$ kpc, $V = 300$ kms^{-1}

$$M_R = 1.98 \times 10^{11} M_\odot$$

(b) $\left(\frac{M_R}{L}\right)_{gal}$ interior to radius R $= \frac{1.98 \times 10^{11}}{3.3 \times 10^9} = 60 \frac{M_\odot}{L_\odot}$

(c) $\left(\frac{M}{L}\right)_* = 6 \left(\frac{M_\odot}{L_\odot}\right)$

Fraction of total mass not in stars

$$= \frac{60-6}{60} = 90\%$$

(d) $M_R = (M_{gas} + M_{dust}) + M_* + M_{dark}$

$$M_{dark} = 1.98 \times 10^{11} - 5 \times 10^9 - 2 \times 10^{10}$$
$$= 1.73 \times 10^{11} M_\odot$$

(e) $\frac{M_{dark}}{M_R} = \frac{1.73}{1.98} = 87.4\%$

AST 358 / HWk 2/ Q 2

(a) $$V_c^2(R)/R = G\,M(R)/R^2$$

Centripetal acceleration = accel. due to force of gravity

$$M(R) = V_c^2(R)\,\frac{R}{G} \qquad (1)$$

At $R = R_h \rightarrow M(R_h) = M_{Tot} = \frac{V_0^2 R_h}{G}$

(b) At $R > R_h$, $M(R)$ in (1) $= M_{Tot}$

$$\rightarrow V_c^2(R) = M(R)\frac{G}{R} = M_{Tot}\frac{G}{R} = V_0^2\,\frac{R_h}{R}$$

$$V_c(R) = V_0\sqrt{\frac{R_h}{R}} \qquad (2)$$

(c) $\Phi(R)$ = Work done in bringing unit mass from ∞ to R

$$= \int_\infty^R F(r)\,dr = \int_\infty^R \frac{G M(r)}{r^2}\,dr$$

$$= \int_\infty^{R_h} \frac{G M(r)}{r^2}\,dr + \int_{R_h}^{R} \frac{G M(r)}{r^2}\,dr \qquad \text{for } R_t \leq R \leq R_h$$

$$= \int_\infty^{R_h} \frac{G M_{Tot}}{r^2}\,dr + \int_{R_h}^{R} \frac{V_c^2(r)}{r}\,dr$$

Use (1) and (2)

$$= V_0^2 R_h \int_\infty^{R_h} \frac{1}{r^2}\,dr + V_0^2 \int_{R_h}^{R} \frac{1}{r}\,dr$$

$$= -V_0^2 R_h \left[\frac{1}{r}\right]_\infty^{R_h} + V_0^2\left[\ln r\right]_{R_h}^{R}$$

$$\rightarrow \Phi(R) = -V_0^2\left[1 + \ln(R_h/R)\right] \quad \text{for } R_t \leq R \leq R_h \qquad (3)$$

(4) Escape speed V_e satisfies

$$\frac{1}{2} V_e^2 = |\Phi(R)| \qquad \text{from conservation of energy} \qquad (4)$$

For $R_t \leq R \leq R_h$ use $\Phi(R)$ from (3)

$$V_e^2(R) = 2V_0^2 \left[1 + \ln(R_h/R) \right]$$

$$\ln(R_h/R) = \frac{1}{2}\left(\frac{V_e(R)}{V_0}\right)^2 - 1 \qquad (5)$$

At $R = R_\odot$, $V_e(R_\odot) \geq (500 \text{ kms}^{-1})^2$

$$\Rightarrow \quad \ln(R_h/R_\odot) \geq \frac{1}{2}\left(\frac{500}{220}\right)^2 - 1$$

$$R_h \geq 4.9 \, R_\odot \text{ or } 41.3 \text{ kpc}$$