ASTRO 358: HOMEWORK 2

Assigned on Th Feb 23, 2012. Due at the start of class on Th Mar 1, 2012

Instructions: In order to get full credit, you must show the method that you used to derive the answer. The number of points for each question is indicated in brackets, and the total score is 100 points.

1. Consider a star in the disk of a spiral galaxy, moving at a speed $v \sim 3 \times 10^5$ m s^{-1} along a circular orbit of radius $R \sim 10$ kpc. Assume for simplicity that the mass distribution can be treated as spherically symmetric.

(a) Calculate the mass $M(R)$ enclosed within radius R, expressing your answer in $M_\odot$. [**5 pts**]

(b) The total stellar luminosity of the galaxy generated within radius R is 3.3×10^9 $L_\odot$. What is the average mass-to-light ratio $(M/L)_{\rm gal}$ of the galaxy within radius R?. [**5 pts**]

(c) Assuming that the average mass-to-light ratio $(M/L)_{\rm star}$ of stellar populations in the disk is ~ 6 $M_\odot/L_\odot$, determine what fraction of the total mass $M(R)$ interior to radius R is *not* in the form of stars? [**10 pts**]

(d) The total mass $M(R)$ interior to radius R is made up of stars, dust, gas, and dark matter. Optical and radio observations show that inside the radius R, the mass of stars is 2×10^{10} $M_\odot$, while the mass of gas and dust amounts to 5×10^9 $M_\odot$. Estimate the mass of dark matter inside the radius R, expressing your answer in $M_\odot$. [**15 pts**].

(e) Determine what fraction of the total mass $M(R)$ interior to radius R is in the form of dark matter. [**10 pts**].

2. Assume that the rotation curve of the Milky Way, namely its circular speed $V_{\rm c}(R)$ as a function of galactocentric radius R, is given by Figure 1 on the next page. On Figure 1, $V_{\rm o}$ is the constant circular speed over the flat part of the rotation curve; $R_{\rm t}$ is the turnover radius of the rotation curve such that $V_{\rm c}(R) = V_{\rm o}$ for $R_{\rm t} \leq R \leq R_{\rm h}$; $R_{\rm h}$ is the radius of the dark matter halo such that the galaxy has no mass at $R > R_{\rm h}$; and $R_\odot$ denotes the solar radius of 8.5 kpc (i.e., the distance of the Sun from the Galactic center).

(a) Derive an expression for the total mass of the Milky Way, assuming that its mass distribution can be considered as spherically symmetric. Express your answer in terms of $V_{\rm o}$ and $R_{\rm h}$. [**10 pts**]

(b) Derive an expression for the circular speed $V_{\rm c}(R)$ of the Milky Way as a function of R at radii $R > R_{\rm h}$, assuming that its mass distribution can be considered as spherically symmetric. [**10 pts**].

(c) Derive expressions for the gravitational potential $\Phi(R)$ at a radius R satisfying $R_{\rm t} \leq R \leq R_{\rm h}$. [**15 pts**]

(d) The fastest stars observed around the solar radius $R_\odot$ have speeds of 500 km s^{-1}. Taking this value as the lower limit to the escape speed at radius $R_\odot$, estimate a lower limit for the radius $R_{\rm h}$ of the dark matter halo. Assume that $R_\odot \sim 8.5$ kpc and $V_{\rm o} = 220$ km s^{-1}. [Hint: Note that $R_{\rm t} \leq R_\odot \leq R_{\rm h}$ and use the results from (c) for $\Phi(R)$]. [**20 pts**]

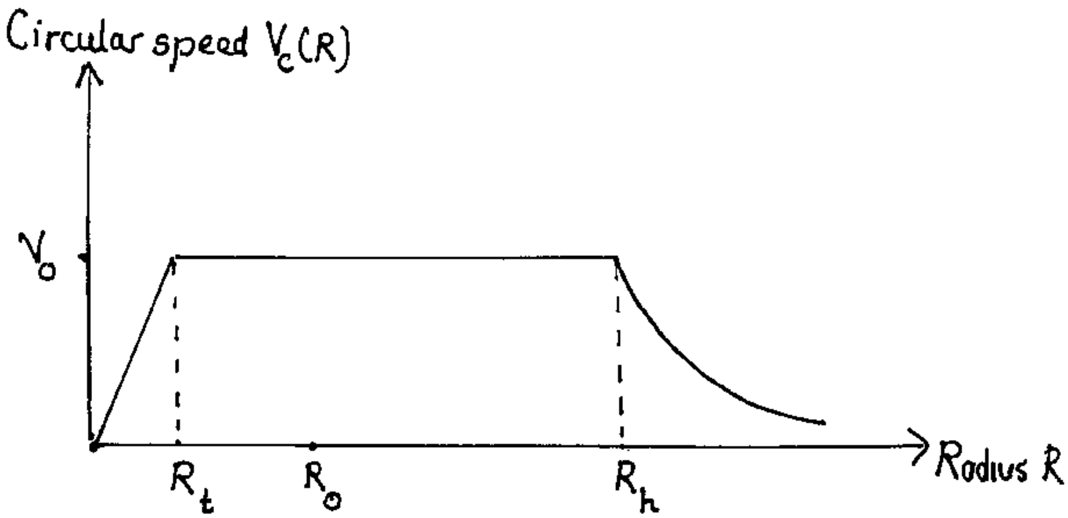

Figure 1 .

Values of physical constants

1 $M_\odot = 2 \times 10^{30}$ kg.
G = Gravitational constant = 6.7×10^{-11} N m^2 kg^{-2} = 4.5×10^{-3} (km s^{-1})^2 pc $M_\odot{}^{-1}$
Wien's constant $W = 2.9 \times 10^{-3}$ m K
Hubble's constant $H_0 = 75$ km s^{-1} Mpc^{-1}
σ = Stefan-Boltzmann constant = 5.7×10^{-8} J s^{-1} m^{-2} K^{-4}

END OF ASSIGNMENT