Hwk 1, Solution Set, A358

1 No of galaxies per unit volume with $L > 0$

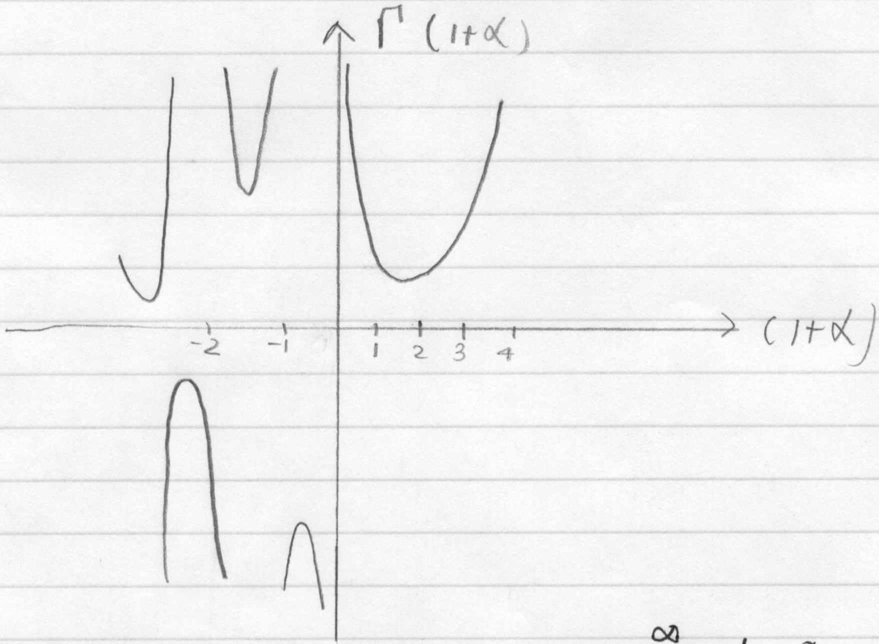
$$= n(L > 0) = \int_0^\infty \Phi(L)\, dL$$

$$= \left(\frac{\Phi^*}{L^*}\right) \int_0^\infty \left(\frac{L}{L^*}\right)^\alpha e^{-(L/L^*)}\, dL$$

$$= \Phi^* \int_0^\infty x^\alpha e^{-x}\, dx \qquad \text{where } x = \frac{L}{L^*}$$

$$= \Phi^* \underbrace{\Gamma(1+\alpha)}_{\text{Gamma function}}$$

= tends to ∞ as α tends to -1
(see graph below)



Plot of Gamma function $\Gamma(1+\alpha) = \int_0^\infty x^\alpha e^{-x}\, dx$

(Ref = Abramowitz)

2 Total luminosity per unit volume

$$= \ell(L>0) = \int_0^\infty \Phi(L)\, L\, dL$$

$$= \Phi^* \int \left(\frac{L}{L^*}\right)\left(\frac{L}{L^*}\right)^\alpha e^{-L/L^*}\, dL$$

$$= \Phi^* L^* \int_0^\infty x^{\alpha+1} e^{-x}\, dx \quad \text{where } x = L/L^*$$

$$= \Phi^* L^* \underbrace{\Gamma(2+\alpha)}_{\text{Gamma function}}$$

$$= \Phi^* L^* \quad \text{for } \alpha = -1.0 \quad (\text{Ref} = \text{Abramowitz,}$$

$$= \text{finite !}$$

3 For a magnitude-limited survey, with a limiting flux f_c

$\rightarrow$ Object with luminosity L are detected out to a maximum distance d_{max} and over a max. volume V_{max}, where

$$\left(f = \frac{L}{4\pi d_{max}^2}\right) = f_c$$

$$d_{max} = \sqrt{L/4\pi f_c} \qquad (1)$$

$$V_{max} = \frac{4}{3}\pi d_{max}^3 = \frac{4}{3}\pi \left(\frac{L}{4\pi f_c}\right)^{3/2} \qquad (2)$$

Use (2) and a Schecter luminosity function: total no of galaxies detected with Luminosity in range L_1 to L_2

$$= N_{12} = \int_{L_1}^{L_2} \Phi(L)\, V_{max}(L)\, dL$$

$$= \int_{L_1}^{L_2} \frac{\Phi^*}{L^*} \left(\frac{L}{L^*}\right)^{\alpha} e^{-L/L^*} \frac{4}{3}\pi \left(\frac{L}{4\pi f_c}\right)^{3/2} dL$$

$$= N_c \int_{L_1}^{L_2} \left(\frac{L}{L^*}\right)^{\alpha+\frac{3}{2}} e^{-L/L^*} \frac{dL}{L^*} \qquad (3)$$

where

$$N_c = \Phi^* \frac{4}{3}\pi \left(\frac{L^*}{4\pi f_c}\right)^{3/2}$$

For $\alpha = -1.5$, (3) reduces to

$$N_{12} = N_c \int_{L_1}^{L_2} e^{-L/L^*} \frac{dL}{L^*} = N_c \int_{x_1}^{x_2} e^{-x}\, dx \qquad (4a)$$

where $x = L/L^*$

$$= N_c \left[e^{-x_1} - e^{-x_2} \right] \qquad (4b)$$

x_1	x_2	N_{12}
0	∞	Total no of galaxies $= N_c$
0.7	∞	No of galaxies with $L > 0.7 L^* = 0.49\, N_c \sim 50\%\, N_c$
3	∞	No . . . . . . $L > 3 L^* = 0.05\, N_c \sim 5\%$
0	0.05	. . . . . . $L < 0.05 L^* = 0.05\, N_c \sim 5\%$.

Homework 1, Question 4

Galaxy Name	Coded Hubble Type	Revised Hubble Type	D_25 [arcmin]	(B-V)_T	V_GSR [km/s]	D [Mpc]
NGC720	.E.5...	E5	4.68	0.98	1715	24.5
NGC4772	.SAS1..	SA(s)a	3.39	0.92	968	13.8
NGC5248	.SXT4..	SAB(rs)bc	6.17	0.65	1128	16.11
SMC	.SBS9P.	SB(s)m pec	316	0.45	34	0.49
M32	CE.2	dE2	8.7	0.95	-28	----

NOTE: Hubble's law describes the relation between the distance of a galaxy and its RECESSION velocity caused by the expansion of the Universe. If the observed object is approaching us (as in the case of M32) rather than receding away from us, then this implies that the gravitational effects of the local mass distribution (rather than the expansion of the Universe) are dominating the motion of the galaxy. Hence, Hubble's law cannot be applied to the case of M32.