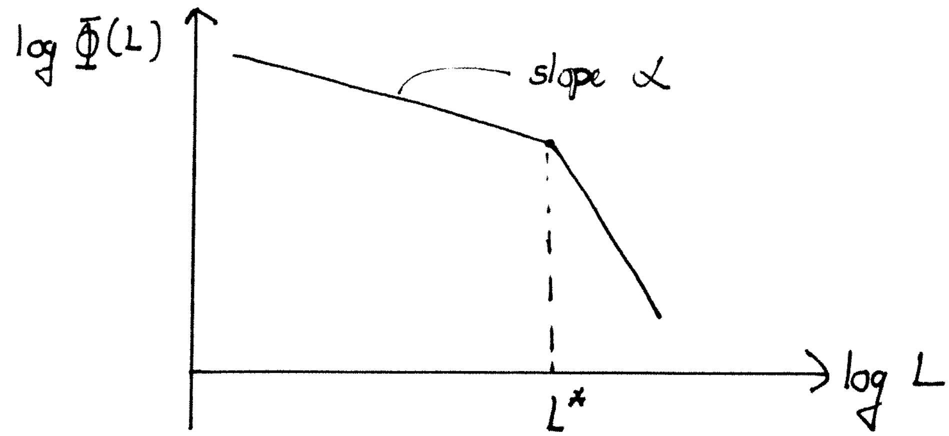
- Q3a



- Q3c

For a magnitude limited survey, objects detected have flux $> f_c$
An object of luminosity L is detected out to distance d_{max}, where

$$f = \frac{L}{4\pi d_{max}^2} = f_c \qquad (1)$$

$$\text{Volume} \quad V_{max}(L) = \frac{4}{3}\pi d_{max}^3 = \left(\frac{L}{4\pi f_c}\right)^{3/2} \frac{4\pi}{3} \qquad (2)$$

Total no of galaxies detected with luminosity $a_1 < L/L^* < a_2$ is

$$N = \int_{a_1}^{a_2} (L)\; V_{max}(L)\; dL$$

$$= \int_{a_1}^{a_2} \phi^*/L^* \left(\frac{L}{L^*}\right)^{\alpha} \exp\left(\frac{-L}{L^*}\right) \left(\frac{L}{4\pi f_c}\right)^{3/2} \frac{4}{3}\pi \; dL$$

$$= \underbrace{\phi^* \left(\frac{L^*}{4\pi f_c}\right)^{3/2} \frac{4\pi}{3}} \exp\left(\frac{-L}{L^*}\right) \frac{dL}{L^*} \qquad \text{for } \alpha = -1.5$$

$$= N_c \int_{a_1}^{a_2} e^{-x}\, dx = N_c \left[e^{-a_1} - e^{-a_2}\right] \qquad \text{where } x = \frac{L}{L^*}$$

For $a_1 = 0 \quad a_2 = \infty \qquad N = N_c$
For $a_1 = 0.7 \quad a_2 = \infty \qquad N = N_c \times \frac{1}{e} \sim N_c\, 0.37$.

Q4b

MOND modifies Newton's second law such that force $\underset{\sim}{F}$ and acceleration $\underset{\sim}{a}$ are related by

$$\underset{\sim}{F} = m \underset{\sim}{a} \underbrace{\mu(x)}_{\text{correction factor}} \qquad (1)$$

where $x = \frac{|\underset{\sim}{a}|}{a_0} = \frac{a}{a_0}$, with $a_0 = $ constant $\qquad (2)$

$$\mu(x) = 1 \quad \text{if} \quad |x| \gg 1 \qquad (3a)$$
$$= x \quad \text{if} \quad |x| \ll 1 \qquad (3b)$$

i.e. We recover Newton's 2nd law at large accelerations, but for very weak accelerations, $\underset{\sim}{F} \ll m\underset{\sim}{a}$

At large radii r in a galaxy, the acceleration is small s.t $x \ll 1$ and $\mu(x) = x$. So

$$\frac{GM(r)}{r^2} = a\,\mu(x)$$
$$= a x = a^2/a_0$$

$$\rightarrow \quad a = \frac{[GM(r)\,a_0]^{1/2}}{r} \qquad (4)$$

Circular speed V_c satisfies

$$V_c^2/r = \text{acceleration } a \text{ due to gravity} \qquad (5)$$

At large r, from (4) it follows

$$V_c^2/r = \frac{[GM(r)\,a_0]^{1/2}}{r}$$

$$\Rightarrow \quad V_c^2 = [GM(r)\,a_0]^{1/2} \qquad (6)$$

At large r, the enclosed mass $M(r)$ rises very slowly and is almost flat with radius, IF all the mass is visible mass and there is no dark matter. This will naturally produce a nearly flat rotation curve from (6)

- Q4c

$$\frac{V_c^2(R)}{R} = \frac{G\,M(R)}{R^2}$$

$$M(R) = \frac{V_c^2(R)\,R}{G} = 222\ M_\odot \left(\frac{V_c}{\mathrm{km\,s^{-1}}}\right)^2 \left(\frac{R}{pc}\right)$$

$$= 222\ M_\odot\ (300)^2\ (30\times10^3)$$

$$= 6.0 \times 10^{11}\ M_\odot$$

= total mass interior to $R - 30$ kpc.

Gas mass $M_{gas}(R) = 5\times10^9\ M_\odot$

Stellar mass $M_*(R) = L_* \times \left(\frac{M}{L}\right) = 10^{10}\times 6\ M_\odot$

Dark matter mass $= M(R) - M_{gas} - M_*$

$$= (600 - 5 - 60) \times 10^9\ M_\odot$$

$$= 5.35 \times 10^{11}\ M_\odot$$

- Q4d

Escape speed $V_e(R)$ satisfies

$$\frac{1}{2} V_e^2(R) = |\phi(R)| \quad \text{conservation of energy}$$

$$V_e(R) = \sqrt{2|\phi(R)|} = \sqrt{2\frac{G\,M(R)}{R}}$$

At $R >$ Radius containing total mass of galaxy M_g

$$V_e(R) = \sqrt{\frac{2GM_g}{R}}$$

$$= \sqrt{\frac{2}{222}\left(\frac{M_g}{M_\odot}\right)\Big/\left(\frac{R}{pc}\right)}\ \mathrm{km\,s^{-1}}$$

$$= \sqrt{\frac{2}{222}\times 6\times10^{11} \times /4\times10^4} = 368\ \mathrm{km\,s^{-1}}$$

Q 5b

Einstein radius $\Theta_E = \sqrt{\left(\frac{M}{M_\odot}\right) 6 \times 10^3 m \left(\frac{d_{LS}}{d_L d_S}\right)}$

d_L = distance to lens
d_S = source
d_{LS} = between source and lens

$d_L = 10$ kpc
For a distant star $d_S >> d_L$, $d_{LS} \simeq d_S$

$$\Theta_E = 9 \times 10^{-4} \text{ arcsec } \sqrt{\left(\frac{M}{M_\odot}\right)\left(\frac{10 \text{ kpc}}{d_L}\right)}$$

For the images at separation $2\Theta_E$ to be resolved by 0.1" image

$$\left(\frac{0.1''}{2}\right)^2 \leq \left(9 \times 10^{-4}\right)^2 \left(\frac{M}{M_\odot}\right)$$

$$M \geq 3.1 \times 10^3 \; M_\odot$$