A. The Electromagnetic Radiation Field

In this appendix, we will briefly review the most important properties of a radiation field. We thereby assume that the reader has encountered these quantities already in a different context.

A.1 Parameters of the Radiation Field

The electromagnetic radiation field is described by the *specific intensity* I_ν, which is defined as follows. Consider a surface element of area $\mathrm{d}A$. The radiation energy which passes through this area per time interval $\mathrm{d}t$ from within a solid angle element $\mathrm{d}\omega$ around a direction described by the unit vector $\boldsymbol{n}$, with frequency in the range between ν and $\nu + \mathrm{d}\nu$, is

$$\mathrm{d}E = I_\nu \, \mathrm{d}A \, \cos\theta \, \mathrm{d}t \, \mathrm{d}\omega \, \mathrm{d}\nu \, , \tag{A.1}$$

where θ describes the angle between the direction $\boldsymbol{n}$ of the light and the normal vector of the surface element. Then, $\mathrm{d}A \, \cos\theta$ is the area projected in the direction of the infalling light. The specific intensity depends on the considered position (and, in time-dependent radiation fields, on time), the direction $\boldsymbol{n}$, and the frequency ν. With the definition (A.1), the dimension of I_ν is energy per unit area, time, solid angle, and frequency, and it is typically measured in units of $\mathrm{erg\,cm^{-2}\,s^{-1}\,ster^{-1}\,Hz^{-1}}$. The specific intensity of a cosmic source describes its surface brightness.

The *specific net flux* F_ν passing through an area element is obtained by integrating the specific intensity over all solid angles,

$$F_\nu = \int \mathrm{d}\omega \, I_\nu \, \cos\theta \, . \tag{A.2}$$

The flux that we receive from a cosmic source is defined in exactly the same way, except that cosmic sources usually subtend a very small solid angle on the sky. In calculating the flux we receive from them, we may therefore drop the factor $\cos\theta$ in (A.2); in this context, the specific flux is also denoted as S_ν. However, in this Appendix (and only here!), the notation S_ν will be reserved for another quantity. The flux is measured in units of $\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}$. If the radiation field is isotropic, F_ν vanishes. In this case, the same amount of radiation passes through the surface element in both directions.

The *mean specific intensity* J_ν is defined as the average of I_ν over all angles,

$$J_\nu = \frac{1}{4\pi} \int \mathrm{d}\omega \, I_\nu \, , \tag{A.3}$$

so that, for an isotropic radiation field, $I_\nu = J_\nu$. The *specific energy density* u_ν is related to J_ν according to

$$u_\nu = \frac{4\pi}{c} J_\nu \tag{A.4}$$

where u_ν is the energy of the radiation field per volume element and frequency interval, thus measured in $\mathrm{erg\,cm^{-3}\,Hz^{-1}}$. The total energy density of the radiation is obtained by integrating u_ν over frequency. In the same way, the intensity of the radiation is obtained by integrating the specific intensity I_ν over ν.

A.2 Radiative Transfer

The specific intensity of radiation in the direction of propagation between source and observer is constant, as long as no emission or absorption processes are occurring. If s measures the length along a line-of-sight, the above statement can be formulated as

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = 0 \, . \tag{A.5}$$

An immediate consequence of this equation is that the surface brightness of a source is independent of its distance. The observed flux of a source depends on its distance, because the solid angle, under which the source is observed, decreases with the square of the distance, $F_\nu \propto D^{-2}$ (see Eq. A.2). However, for light propagating through a medium, emission and absorption (or scattering of light) occurring along the path over which the light travels may change the specific intensity. These effects are described by the *equation of radiative transfer*

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu \, I_\nu + j_\nu \, . \tag{A.6}$$

The first term describes the absorption of radiation and states that the radiation absorbed within a length interval $\mathrm{d}s$ is proportional to the incident radiation.

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The factor of proportionality is the *absorption coefficient* κ_ν, which has the unit of cm^{-1}. The *emission coefficient* j_ν describes the energy that is added to the radiation field by emission processes, having a unit of $\mathrm{erg\,cm^{-3}\,s^{-1}\,Hz^{-1}\,ster^{-1}}$; hence, it is the radiation energy emitted per volume element, time interval, frequency interval, and solid angle. Both, κ_ν and j_ν depend on the nature and state (such as temperature, chemical composition) of the medium through which light propagates.

The absorption and emission coefficients both account for true absorption and emission processes, as well as the scattering of radiation. Indeed, the scattering of a photon can be considered as an absorption that is immediately followed by an emission of a photon.

The *optical depth* τ_ν along a line-of-sight is defined as the integral over the absorption coefficient,

$$\tau_\nu(s) = \int_{s_0}^{s} \mathrm{d}s'\, \kappa_\nu(s') \,, \tag{A.7}$$

where s_0 denotes a reference point on the sightline from which the optical depth is measured. Dividing (A.6) by κ_ν and using the relation $\mathrm{d}\tau_\nu = \kappa_\nu\,\mathrm{d}s$ in order to introduce the optical depth as a new variable along the light ray, the equation of radiative transfer can be written as

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = -I_\nu + S_\nu \,, \tag{A.8}$$

where the source function

$$S_\nu = \frac{j_\nu}{\kappa_\nu} \tag{A.9}$$

is defined as the ratio of the emission and absorption coefficients. In this form, the equation of radiative transport can be formally solved; as can easily be tested by substitution, the solution is

$$I_\nu(\tau_\nu) = I_\nu(0)\,\exp(-\tau_\nu) + \int_0^{\tau_\nu} \mathrm{d}\tau'_\nu\, \exp\left(\tau'_\nu - \tau_\nu\right)\, S_\nu(\tau'_\nu) \,. \tag{A.10}$$

This equation has a simple interpretation. If $I_\nu(0)$ is the incident intensity, it will have decreased by absorption to a value $I_\nu(0)\,\exp(-\tau_\nu)$ after an optical depth of τ_ν. On the other hand, energy is added to the radiation field by emission, accounted for by the τ'-integral. Only a fraction $\exp\left(\tau'_\nu - \tau_\nu\right)$ of this additional energy emitted at τ' reaches the point τ, the rest is absorbed.

In the context of (A.10), we call this a *formal* solution for the equation of radiative transport. The reason for this is based on the fact that both the absorption and the emission coefficient depend on the physical state of the matter through which radiation propagates, and in many situations this state depends on the radiation field itself. For instance, κ_ν and j_ν depend on the temperature of the matter, which in turn depends, by heating and cooling processes, on the radiation field to which it is exposed. Hence, one needs to solve a coupled system of equations in general: on the one hand the equation of radiative transport, and on the other hand the equation of state for matter. In many situations, very complex problems arise from this, but we will not consider them further in the context of this book.

A.3 Blackbody Radiation

For matter in thermal equilibrium, the source function S_ν is solely a function of the matter temperature,

$$S_\nu = B_\nu(T) \,, \text{ or } j_\nu = B_\nu(T)\,\kappa_\nu \,, \tag{A.11}$$

independent of the composition of the medium (Kirchhoff's law). We will now consider radiation propagating through matter in thermal equilibrium at constant temperature T. Since in this case $S_\nu = B_\nu(T)$ is constant, the solution (A.10) can be written in the form

$$\begin{aligned} I_\nu(\tau_\nu) &= I_\nu(0)\,\exp(-\tau_\nu) + B_\nu(T) \int_0^{\tau_\nu} \mathrm{d}\tau'_\nu\, \exp\left(\tau'_\nu - \tau_\nu\right) \\ &= I_\nu(0)\,\exp(-\tau_\nu) + B_\nu(T)\left[1 - \exp(-\tau_\nu)\right] \,. \end{aligned} \tag{A.12}$$

From this it follows that $I_\nu = B_\nu(T)$ is valid for sufficiently large optical depth τ_ν. The radiation propagating through matter which is in thermal equilibrium is described by the function $B_\nu(T)$ if the optical depth is sufficiently large, independent of the composition of the matter. A specific case of this situation can be illustrated by imagining the radiation field inside a box whose opaque walls are kept at a constant temperature T. Due to the opaqueness of the walls, their optical depth is infinite, hence the radiation field within the box is given by $I_\nu = B_\nu(T)$. This is also valid if the volume is filled with matter, as long as the latter is in thermal equilibrium at temperature T. For these reasons, this kind of radiation field is also called blackbody radiation.

The function $B_\nu(T)$ was first obtained in 1900 by Max Planck, and in his honor, it was named the *Planck function*; it reads

$$B_\nu(T) = \frac{2h_P\nu^3}{c^2}\frac{1}{e^{h_P\nu/k_B T} - 1}\,, \tag{A.13}$$

where $h_P = 6.625 \times 10^{-27}$ erg s is the *Planck constant* and $k_B = 1.38 \times 10^{-16}$ erg K^{-1} is the Boltzmann constant. The shape of the spectrum can be derived from statistical physics. *Blackbody radiation* is defined by $I_\nu = B_\nu(T)$, and *thermal radiation* by $S_\nu = B_\nu(T)$. For large optical depths, thermal radiation converges to blackbody radiation.

The Planck function has its maximum at

$$\frac{h_P\nu_{max}}{k_B T} \approx 2.82\,, \tag{A.14}$$

i.e., the frequency of the maximum is proportional to the temperature. This property is called *Wien's law*. This law can also be written in more convenient units,

$$\nu_{max} = 5.88 \times 10^{10}\,\mathrm{Hz}\,\frac{T}{1\,\mathrm{K}}\,. \tag{A.15}$$

The Planck function can also be formulated depending on wavelength $\lambda = c/\nu$, such that $B_\lambda(T)\,d\lambda = B_\nu(T)\,d\nu$,

$$B_\lambda(T) = \frac{2h_P c^2/\lambda^5}{\exp(h_P c/\lambda k_B T) - 1}\,. \tag{A.16}$$

Two limiting cases of the Planck function are of particular interest. For low frequencies, $h_P\nu \ll k_B T$, one can apply the expansion of the exponential function for small arguments in (A.13). The leading-order term in this expansion then yields

$$B_\nu(T) \approx B_\nu^{RJ}(T) = \frac{2\nu^2}{c^2}\,k_B T\,, \tag{A.17}$$

which is called the *Rayleigh–Jeans approximation* of the Planck function. We point out that the Rayleigh–Jeans equation does not contain the Planck constant, and this law had been known even before Planck derived his exact equation. In the other limiting case of very high frequencies, $h_P\nu \gg k_B T$, the exponential factor in the denominator in (A.13) becomes very much larger than unity, so that we obtain

$$B_\nu(T) \approx B_\nu^{W}(T) = \frac{2h_P\nu^3}{c^2}\,e^{-h_P\nu/k_B T}\,, \tag{A.18}$$

called the *Wien approximation* of the Planck function.

The energy density of blackbody radiation depends only on the temperature, of course, and is calculated by

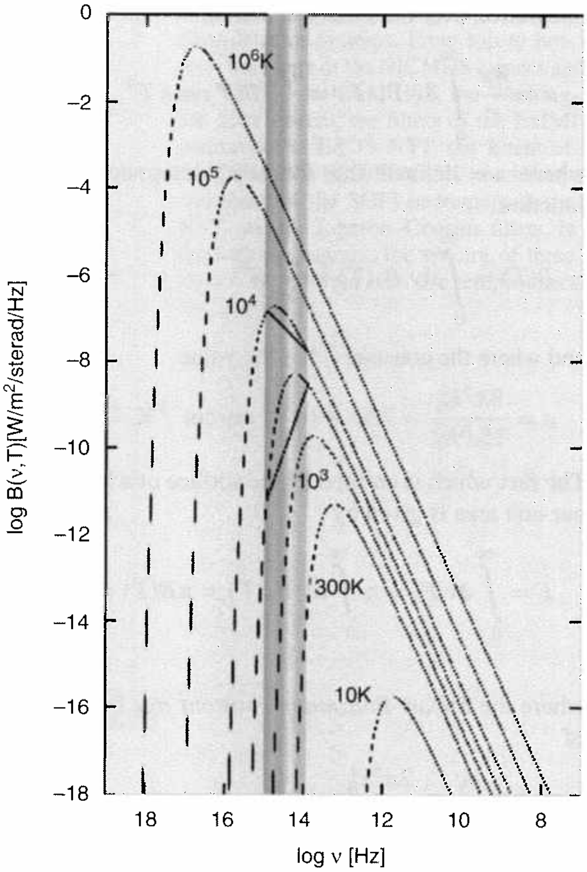

Fig. A.1. The Planck function (A.13) for different temperatures T. The plot shows $B_\nu(T)$ as a function of frequency ν, where high frequencies are plotted towards the left (thus large wavelengths towards the right). The exponentially decreasing Wien part of the spectrum is visible on the left, the Rayleigh–Jeans part on the right. The *shape* of the spectrum in the Rayleigh–Jeans part is independent of the temperature, which is determining the amplitude however integration over the Planck function,

$$u = \frac{4\pi}{c} \int_0^\infty \mathrm{d}\nu \, B_\nu(T) = \frac{4\pi}{c} \, B(T) = a\,T^4 \,, \qquad \text{(A.19)}$$

where we defined the frequency-integrated Planck function

$$B(T) = \int_0^\infty \mathrm{d}\nu \, B_\nu(T) = \frac{a\,c}{4\pi}\,T^4 \,, \qquad \text{(A.20)}$$

and where the constant a has the value

$$a = \frac{8\pi^5 k_\mathrm{B}^4}{15 c^3 h_\mathrm{P}^3} = 7.56 \times 10^{-15}\,\mathrm{erg\,cm^{-3}\,K^{-4}} \,. \qquad \text{(A.21)}$$

The flux which is emitted by the surface of a blackbody per unit area is given by

$$F = \int_0^\infty \mathrm{d}\nu \, F_\nu = \pi \int_0^\infty \mathrm{d}\nu \, B_\nu(T) = \pi B(T) = \sigma_\mathrm{SB} T^4 \,, \qquad \text{(A.22)}$$

where the *Stefan–Boltzmann constant* σ_SB has a value of

$$\sigma_\mathrm{SB} = \frac{ac}{4} = \frac{2\pi^5 k_\mathrm{B}^4}{15 c^2 h_\mathrm{P}^3} = 5.67 \times 10^{-5}\,\mathrm{erg\,cm^{-2}\,K^{-4}\,s^{-1}} \,. \qquad \text{(A.23)}$$

A.4 The Magnitude Scale

Optical astronomy was being conducted well before methods of quantitative measurements became available. The brightness of stars had been cataloged more than 2000 years ago, and their observation goes back as far as the ancient world. Stars were classified into magnitudes, assigning a magnitude of 1 to the brightest stars and higher magnitudes to the fainter ones. Since the apparent magnitude as perceived by the human eye scales roughly logarithmically with the radiation flux (which is also the case for our hearing), the magnitude scale represents a logarithmic flux scale. To link these visually determined magnitudes in historical catalogs to a quantitative measure, the magnitude system has been retained in optical astronomy, although with a precise definition. Since no historical astronomical observations have been conducted in other wavelength ranges, because these are not accessible to the unaided eye, only optical astronomy has to bear the historical burden of the magnitude system.

A.4.1 Apparent Magnitude

We start with a relative system of flux measurements by considering two sources with fluxes S_1 and S_2. The *apparent magnitudes* of the two sources, m_1 and m_2, then behave according to

$$m_1 - m_2 = -2.5 \log\left(\frac{S_1}{S_2}\right) \,; \quad \frac{S_1}{S_2} = 10^{-0.4(m_1 - m_2)} \,. \qquad \text{(A.24)}$$

This means that the brighter source has a smaller apparent magnitude than the fainter one: the larger the apparent magnitude, the fainter the source.[1] The factor of 2.5 in this definition is chosen so as to yield the best agreement of the magnitude system with the visually determined magnitudes. A difference of $|\Delta m| = 1$ in this system corresponds to a flux ratio of ~ 2.51, and a flux ratio of a factor 10 or 100 corresponds to 2.5 or 5 magnitudes, respectively.

A.4.2 Filters and Colors

Since optical observations are performed using a combination of a filter and a detector system, and since the flux ratios depend, in general, on the choice of the filter (because the spectral energy distribution of the sources may be different), apparent magnitudes are defined for each of these filters. The most common filters are shown in Fig. A.2 and listed in Table A.1, together with their characteristic wavelengths and the widths of their transmission curves. The apparent magnitude for a filter X is defined as m_X, frequently written as X. Hence, for the B-band filter, $m_B \equiv B$.

Next, we need to specify how the magnitudes measured in different filters are related to each other, in order to define the color indices of sources. For this

[1] Of course, this convention is confusing, particularly to someone just becoming familiar with astronomy, and it frequently causes confusion and errors, as well as problems in the communication with non-astronomers – but we have to get along with that.

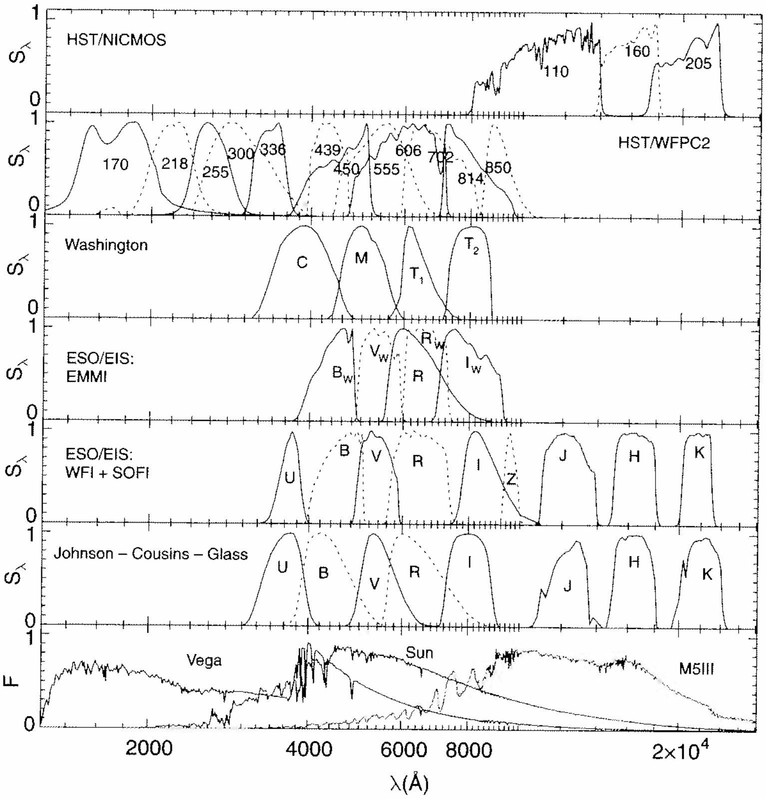

Fig. A.2. Transmission curves of various filter-detector systems. From top to bottom: the filters of the NICMOS camera and the WFPC2 on-board HST, the Washington filter system, the filters of the EMMI instrument at ESO's NTT, the filters of the WFI at the ESO/MPG 2.2-m telescope and those of the SOFI instrument at the NTT, and the Johnson–Cousins filters. In the bottom diagram, the spectra of three stars with different effective temperatures are displayed

Table A.1. For some of the best-established filter systems – Johnson, Strömgren, and the filters of the Sloan Digital Sky Surveys – the central (more precisely, the effective) wavelengths and the widths of the filters are listed

Johnson	U	B	V	R	I	J	H	K	L	M
λ_{eff} (nm)	367	436	545	638	797	1220	1630	2190	3450	4750
$\Delta\lambda$ (nm)	66	94	85	160	149	213	307	39	472	460

Strömgren	u	v	b	y	β_w	β_n
λ_{eff} (nm)	349	411	467	547	489	489
$\Delta\lambda$ (nm)	30	19	18	23	15	3

SDSS	u′	g′	r′	i′	z′
λ_{eff} (nm)	354	477	623	762	913
$\Delta\lambda$ (nm)	57	139	138	152	95

purpose, a particular class of stars is used, main-sequence stars of spectral type A0, of which the star Vega is an archetype. For such a star, by definition, $U = B = V = R = I = \dots$, i.e., every color index for such a star is defined to be zero.

For a more precise definition, let $T_X(\nu)$ be the transmission curve of the filter-detector system. $T_X(\nu)$ specifies which fraction of the incoming photons with frequency ν are registered by the detector. The apparent magnitude of a source with spectral flux S_ν is then

$$m_X = -2.5 \log \left(\frac{\int d\nu \, T_X(\nu) \, S_\nu}{\int d\nu \, T_X(\nu)} \right) + \text{const.} \,, \quad \text{(A.25)}$$

where the constant needs to be determined from reference stars.

Another commonly used definition of magnitudes is the AB system. In contrast to the Vega magnitudes, no stellar spectral energy distribution is used as a reference here, but instead one with a constant flux at all frequencies, $S_\nu^{\text{ref}} = S_\nu^{\text{AB}} = 2.89 \times 10^{-21}\,\text{erg}\,\text{s}^{-1}\,\text{cm}^{-2}\,\text{Hz}^{-1}$. This value has been chosen such that A0 stars like Vega have the same magnitude in the original Johnson V-band as they have in the AB system, $m_V^{\text{AB}} = m_V$. With (A.25), one obtains for the conversion between the two systems

$$m_X^{\text{AB}} - m_X^{\text{Vega}} = -2.5 \log \left(\frac{\int d\nu \, T_X(\nu) \, S_\nu^{\text{AB}}}{\int d\nu \, T_X(\nu) \, S_\nu^{\text{Vega}}} \right) =: m_{\text{AB}\to\text{Vega}} \,. \quad \text{(A.26)}$$

For the filters at the ESO Wide-Field Imager, which are designed to resemble the Johnson set of filters, the following prescriptions are then to be applied: $U_{\text{AB}} = U_{\text{Vega}} + 0.80$; $B_{\text{AB}} = B_{\text{Vega}} - 0.11$; $V_{\text{AB}} = V_{\text{Vega}}$; $R_{\text{AB}} = R_{\text{Vega}} + 0.19$; $I_{\text{AB}} = I_{\text{Vega}} + 0.59$.

A.4.3 Absolute Magnitude

The apparent magnitude of a source does not in itself tell us anything about its luminosity, since for the determination of the latter we also need to know its distance D in addition to the radiative flux. Let L_ν be the specific luminosity of a source, i.e., the energy emitted per unit time and per unit frequency interval, then the flux is given by (note that from here on we switch back to the notation where S denotes the flux, which was denoted by F earlier in this appendix)

$$S_\nu = \frac{L_\nu}{4\pi D^2} \,, \quad \text{(A.27)}$$

where we implicitly assumed that the source emits isotropically. Having the apparent magnitude as a measure for S_ν (at the frequency ν defined by the filter which is applied), it is desirable to have a similar measure for L_ν, specifying the physical properties of the source itself. For this purpose, the *absolute magnitude* is introduced, denoted as M_X, where X refers to the filter under consideration. By definition, M_X is equal to the apparent magnitude of a source if it were to be located at a distance of 10 pc from us. The absolute magnitude of a source is thus independent of its distance, in contrast to the apparent magnitude. With (A.27) we find for the relation of apparent to absolute magnitude

$$m_X - M_X = 5 \log \left(\frac{D}{1\,\text{pc}} \right) - 5 \equiv \mu \,, \quad \text{(A.28)}$$

where we have defined the *distance modulus* μ in the final step. Hence, the latter is a logarithmic measure of the distance of a source: $\mu = 0$ for $D = 10$ pc, $\mu = 10$ for $D = 1$ kpc, and $\mu = 25$ for $D = 1$ Mpc. The difference between apparent and absolute magnitude is independent of the filter choice, and it equals the distance modulus if no extinction is present. In general, this difference is modified by the filter-dependent extinction coefficient – see Sect. 2.2.4.

A.4.4 Bolometric Parameters

The total luminosity L of a source is the integral of the specific luminosity L_ν over all frequencies. Accordingly, the total flux S of a source is the frequency-integrated specific flux S_ν. The *apparent bolometric magnitude* m_{bol} is defined as a logarithmic measure of the total flux,

$$m_{\text{bol}} = -2.5 \log S + \text{const.} \,, \quad \text{(A.29)}$$

where here the constant is also determined from reference stars. Accordingly, the *absolute bolometric magnitude* is defined by means of the distance modulus, as in (A.28). The absolute bolometric magnitude depends on the bolometric luminosity L of a source via

$$M_{\rm bol} = -2.5 \log L + {\rm const.} \qquad (A.30)$$

The constant can be fixed, e.g., by using the parameters of the Sun: its apparent bolometric magnitude is $m_{\odot\rm bol} = -26.83$, and the distance of one Astronomical Unit corresponds to a distance modulus of $\mu = -31.47$. With these values, the absolute bolometric magnitude of the Sun becomes

$$M_{\odot\rm bol} = m_{\odot\rm bol} - \mu = 4.74 \,, \qquad (A.31)$$

so that (A.30) can be written as

$$M_{\rm bol} = 4.74 - 2.5 \log\left(\frac{L}{L_\odot}\right) , \qquad (A.32)$$

and the luminosity of the Sun is then

$$L_\odot = 3.85 \times 10^{33}\,{\rm erg\,s^{-1}} \,. \qquad (A.33)$$

The direct relation between bolometric magnitude and luminosity of a source can hardly be exploited in practice, because the apparent bolometric magnitude (or the flux S) of a source cannot be observed in most cases. For observations of a source from the ground, only a limited window of frequencies is accessible. Nevertheless, in these cases one also likes to quantify the total luminosity of a source. For sources for which the spectrum is assumed to be known, like for many stars, the flux from observations at optical wavelengths can be extrapolated to larger and smaller wavelengths, and so $m_{\rm bol}$ can be estimated. For galaxies or AGNs, which have a much broader spectral distribution and which show much more variation between the different objects, this is not feasible. In these cases, the flux of a source in a particular frequency range is compared to the flux the Sun would have at the same distance and in the same spectral range. If M_X is the absolute magnitude of a source measured in the filter X, the X-band luminosity of this source is defined as

$$L_X = 10^{-0.4(M_X - M_{\odot X})}\, L_{\odot X} \,. \qquad (A.34)$$

Thus, when speaking of, say, the "blue luminosity of a galaxy", this is to be understood as defined in (A.34).

B. Properties of Stars

In this appendix, we will summarize the most important properties of stars as they are required for understanding the contents of this book. Of course, this brief overview cannot replace the study of other textbooks in which the physics of stars is covered in much more detail.

B.1 The Parameters of Stars

To a good approximation, stars are gas spheres, in the cores of which light atomic nuclei are transformed into heavier ones (mainly hydrogen into helium) by thermonuclear processes, thereby producing energy. The external appearance of a star is predominantly characterized by its radius R and its characteristic temperature T. The properties of a star depend mainly on its mass M.

In a first approximation, the spectral energy distribution of the emission from a star can be described by a blackbody spectrum. This means that the specific intensity I_ν is given by a Planck spectrum (A.13) in this approximation. The luminosity L of a star is the energy radiated per unit time. If the spectrum of star was described by a Planck spectrum, the luminosity would depend on the temperature and on the radius according to

$$L = 4\pi R^2 \sigma_{\rm SB} T^4 , \tag{B.1}$$

where (A.22) was applied. However, the spectra of stars deviate from that of a blackbody (see Fig. 3.47). One defines the *effective temperature* $T_{\rm eff}$ of a star as the temperature a blackbody of the same radius would need to have to emit the same luminosity as the star, thus

$$\sigma_{\rm SB} T_{\rm eff}^4 \equiv \frac{L}{4\pi R^2} . \tag{B.2}$$

The luminosities of stars cover a huge range; the weakest are a factor $\sim 10^4$ times less luminous than the Sun, whereas the brightest emit $\sim 10^5$ times as much energy per unit time as the Sun. This big difference in luminosity is caused either by a variation in radius or by different temperatures. We know from the colors of stars that they have different temperatures: there are blue stars which are considerably hotter than the Sun, and red stars that are very much cooler. The temperature of a star can be estimated from its color. From the flux ratio at two different wavelengths or, equivalently, from the color index $X - Y \equiv m_X - m_Y$ in two filters X and Y, the temperature $T_{\rm c}$ is determined such that a blackbody at $T_{\rm c}$ would have the same color index. $T_{\rm c}$ is called the *color temperature* of a star. If the spectrum of a star was a Planck spectrum, then the equality $T_{\rm c} = T_{\rm eff}$ would hold, but in general these two temperatures differ.

B.2 Spectral Class, Luminosity Class, and the Hertzsprung–Russell Diagram

The spectra of stars can be classified according to the atomic (and, in cool stars, also molecular) spectral lines that are present. Based on the line strengths and their ratios, the Harvard sequence of stellar spectra was introduced. These spectral classes follow a sequence that is denoted by the letters O, B, A, F, G, K, M; besides these, some other spectral classes exist that will not be mentioned here. The sequence corresponds to a sequence of color temperature of stars: O stars are particularly hot, around 50 000 K, M stars very much cooler with $Tc \sim 3500$ K. For a finer classification, each spectral class is supplemented by a number between 0 and 9. An A1 star has a spectrum very similar to that of an A0 star, whereas an A5 star has as many features in common with an A0 star as with an F0 star.

Plotting the spectral type versus the absolute magnitude for those stars for which the distance and hence the absolute magnitude can be determined, a striking distribution of stars becomes apparent in such a *Hertzsprung–Russell diagram* (HRD). Instead of the spectral class, one may also plot the color index of the stars, typically $B - V$ or $V - I$. The resulting *color–magnitude diagram* (CMD) is essentially equivalent to an HRD, but is based solely on photometric data. A different but very similar diagram plots the luminosity versus the effective temperature.

In Fig. B.1, a color–magnitude diagram is plotted, compiled from data observed by the HIPPARCOS satellite. Instead of filling the two-dimensional parameter space rather uniformly, characteristic regions exist in

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DOI: 10.1007/11614371_B

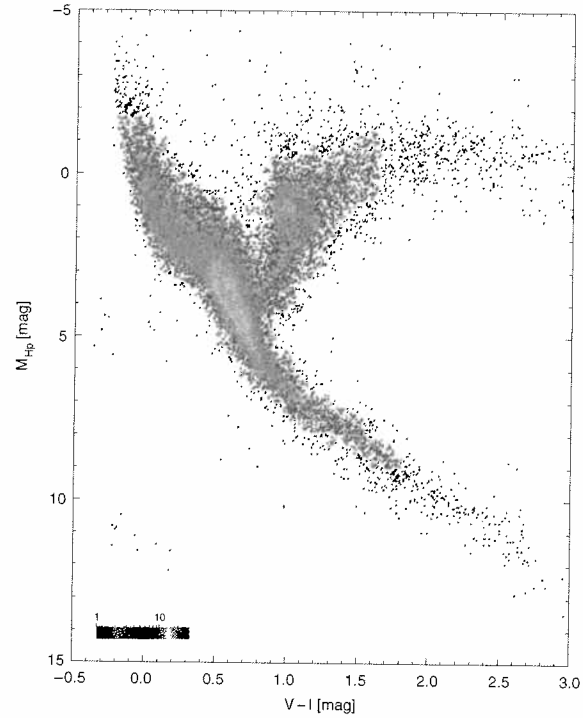

Fig. B.1. Color–magnitude diagram for 41 453 individual stars, whose parallaxes were determined by the HIPPARCOS satellite with an accuracy of better than 20%. Since the stars shown here are subject to unavoidable strong selection effects favoring nearby and luminous stars, the relative number density of stars is not representative of their true abundance. In particular, the lower main sequence is much more densely populated than is visible in this diagram

such color–magnitude diagrams in which nearly all stars are located. Most stars can be found in a thin band called the *main sequence*. It extends from early spectral types (O, B) with high luminosities ("top left") down to late spectral types (K, M) with low luminosities ("bottom right"). Branching off from this main sequence towards the "top right" is the domain of red giants, and below the main sequence, at early spectral types and very much lower luminosities than on the main sequence itself, we have the domain of white dwarfs. The fact that most stars are arranged along a one-dimensional sequence – the main sequence – is probably one of the most important discoveries in astronomy, because it tells us that the properties of stars are determined basically by a single parameter: their mass.

Since stars exist which have, for the same spectral type and hence the same color temperature (and roughly the same effective temperature), very different luminosities, we can deduce immediately that these stars have different radii, as can be read from (B.2). Therefore, stars on the red giant branch, with their much higher luminosities compared to main-sequence stars of the same spectral class, have a very much larger radius than the corresponding main-sequence stars. This size effect is also observed spectroscopically: the gravitational acceleration on the surface of a star (surface gravity) is

$$g = \frac{GM}{R^2} \; . \tag{B.3}$$

We know from models of stellar atmospheres that the width of spectral lines depends on the gravitational acceleration on the star's surface: the lower the surface gravity, the narrower the stellar absorption lines. Hence, a relation exists between the line width and the stellar radius. Since the radius of a star – for a fixed spectral type or effective temperature – specifies the luminosity, this luminosity can be derived from the width of the lines. In order to calibrate this relation, stars of known distance are required.

Based on the width of spectral lines, stars are classified into *luminosity classes*: stars of luminosity class I are called supergiants, those of luminosity class III are giants, main-sequence stars are denoted as dwarfs and belong to luminosity class V; in addition, the classification can be further broken down into bright giants (II), subgiants (IV), and subdwarfs (VI). Any star in the Hertzsprung–Russell diagram can be assigned a luminosity class and a spectral class (Fig. B.2). The Sun is a G2 star of luminosity class V.

If the distance of a star, and thus its luminosity, is known, and if in addition its surface gravity can be derived from the line width, we obtain the stellar mass from these parameters. By doing so, it turns out that for main-sequence stars the luminosity is a steep function of the stellar mass, approximately described by

$$\frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot} \right)^{3.5} \; . \tag{B.4}$$

Therefore, a main-sequence star of $M = 10 M_\odot$ is ~ 3000 times more luminous than our Sun.

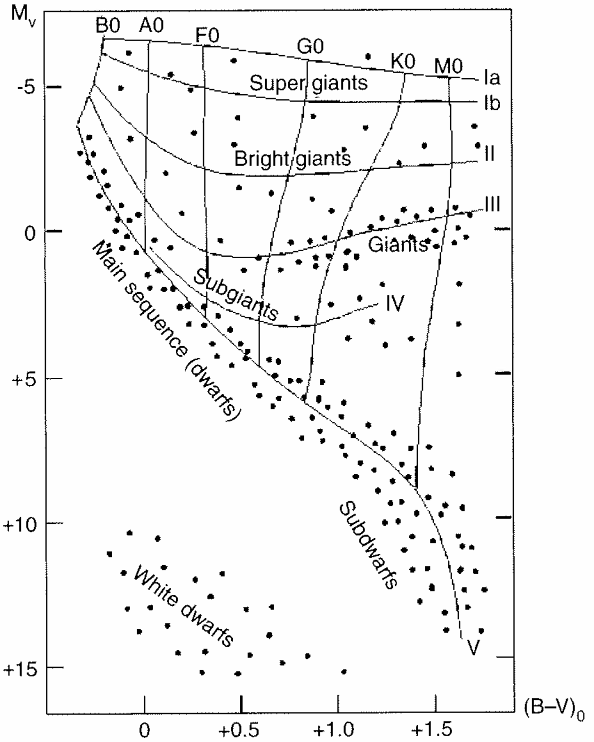

Fig. B.2. Schematic color–magnitude diagram in which the spectral types and luminosity classes are indicated

B.3 Structure and Evolution of Stars

To a very good approximation, stars are spherically symmetric. Therefore, the structure of a star is described by the radial profile of the parameters of its stellar plasma. These are density, pressure, temperature, and chemical composition of the matter. During almost the full lifetime of a star, the plasma is in hydrostatic equilibrium, so that pressure forces and gravitational forces are of equal magnitude and directed in opposite directions, so as to balance each other.

The density and temperature are sufficiently high in the center of a star that thermonuclear reactions are ignited. In main-sequence stars, hydrogen is fused into helium, thus four protons are combined into one ^{4}He nucleus. For every helium nucleus that is produced this way, 26.73 MeV of energy are released. Part of this energy is emitted in the form of neutrinos which can escape unobstructed from the star due to their very low cross-section.[1] The energy production rate is approximately proportional to T^4 for temperatures below about 15×10^6 K, at which the reaction follows the so-called pp-chain. At higher temperatures, another reaction chain starts to contribute, the so-called CNO cycle, with an energy production rate which is much more strongly dependent on temperature – roughly proportional to T^{20}.

The energy generated in the interior of a star is transported outwards, where it is then released in the form of electromagnetic radiation. This energy transport may take place in two different ways: first, by radiation transport, and second, it can be transported by macroscopic flows of the stellar plasma. This second mechanism of energy transport is called convection; here, hot elements of the gas rise upwards, driven by buoyancy, and at the same time cool ones sink downwards. The process is similar to that observed in heating water on a stove. Which of the two processes is responsible for the energy transport depends on the temperature profile inside the star. The intervals in a star's radius in which energy transport takes place via convection are called convection zones. Since in convection zones stellar material is subject to mixing, the chemical composition is homogeneous there. In particular, chemical elements produced by nuclear fusion are transported through the star by convection.

Stars begin their lives with a homogeneous chemical composition, resulting from the composition of the molecular cloud out of which they are formed. If their mass exceeds about $0.08 M_\odot$, the temperature and pressure in their core are sufficient to ignite the fusion of hydrogen into helium. Gas spheres with a mass below $\sim 0.08 M_\odot$ will not satisfy these conditions, hence these objects – they are called brown dwarfs – are not stars in

[1] The detection of neutrinos from the Sun in terrestrial detectors was the final proof for the energy production mechanism being nuclear fusion. However, the measured rate of electron neutrinos from the Sun was only half as large as expected from Solar models. This *Solar neutrino problem* kept physicists and astrophysicists busy for decades. It was a first indication of neutrinos having a finite rest mass – only in this case could electron neutrinos transform into another sort of neutrino along the way from the Sun to us. Recently, these neutrino oscillations were confirmed: neutrinos have a very small but finite rest mass. For their research in the field of Solar neutrinos, Raymond Davis and Masatoshi Koshiba were awarded with one half of the Nobel Prize in Physics in 2002. The other half was awarded to Ricardo Giacconi for his pioneering work in the field of X-ray astronomy.

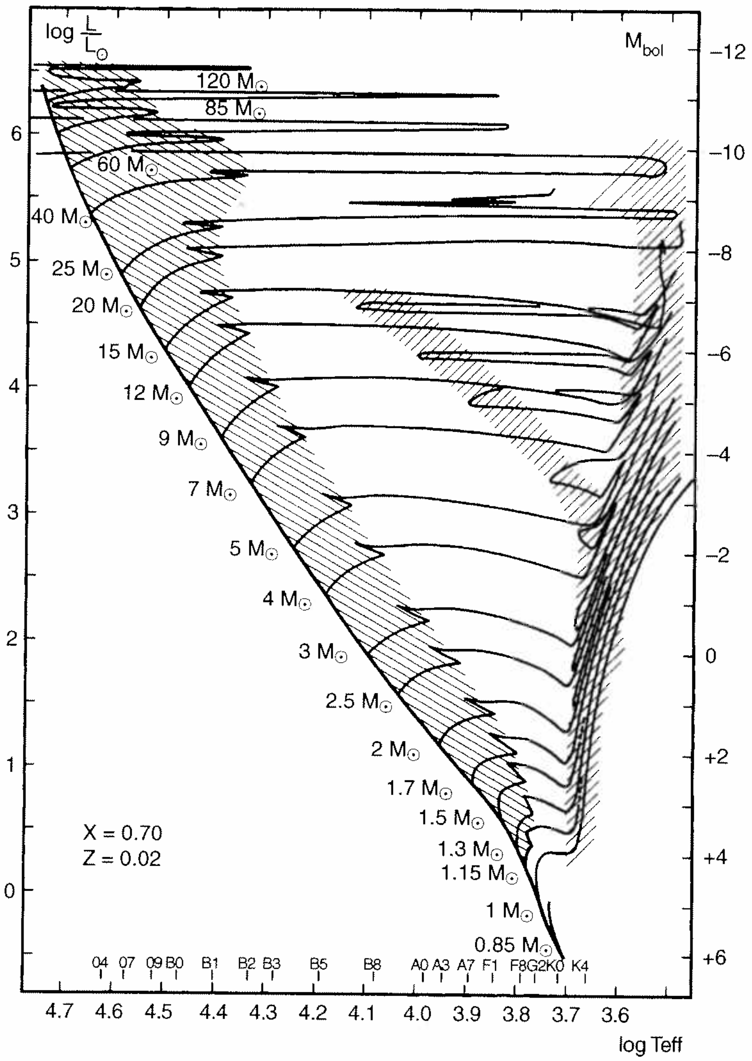

Fig. B.3. Theoretical temperature-luminosity diagram of stars. The solid curve is the zero age main sequence (ZAMS), on which stars ignite the burning of hydrogen in their cores. The evolutionary tracks of these stars are indicated by the various lines which are labeled with the stellar mass. The hatched areas mark phases in which the evolution proceeds only slowly, so that many stars are observed to be in these areas

a proper sense.[2] At the onset of nuclear fusion, the star is located on the zero-age main sequence (ZAMS) in the HRD (see Fig. B.3). The energy production by fusion of hydrogen into helium alters the chemical composition in the stellar interior; the abundance of hydrogen decreases by the same rate as the abundance of helium increases. As a consequence, the duration of this phase of central hydrogen burning is limited. As a rough estimate, the conditions in a star will change noticeably when about 10% of its hydrogen is used up. Based on this criterion, the lifetime of a star on the main sequence can now be estimated. The total energy produced in this phase can be written as

$$E_{\rm MS} = 0.1 \times Mc^2 \times 0.007\,, \tag{B.5}$$

[2] If the mass of a brown dwarf exceeds $\sim 0.013 M_\odot$, the central density and temperature are high enough to enable the fusion of deuterium (heavy hydrogen) into helium. However, the abundance of deuterium is smaller by several orders of magnitude than that of normal hydrogen, rendering the fuel reservoir of a brown dwarf very small.

where Mc^2 is the rest-mass energy of the star, of which a fraction of 0.1 is fused into helium, which is supposed to occur with an efficiency of 0.007. Phrased differently, in the fusion of four protons into one helium nucleus, an energy of $\sim 0.007 \times 4 m_{\rm p} c^2$ is generated, with $m_{\rm p}$ denoting the proton mass. In particular, (B.5) states that the total energy produced during this main-sequence phase is proportional to the mass of the star. In addition, we know from (B.4) that the luminosity is a steep function of the stellar mass. The lifetime of a star on the main sequence can then be estimated by equating the available energy $E_{\rm MS}$ with the product of luminosity and lifetime. This yields

$$t_{\rm MS} = \frac{E_{\rm MS}}{L} \approx 8 \times 10^9 \, \frac{M/M_\odot}{L/L_\odot} \, \text{yr} \approx 8 \times 10^9 \left(\frac{M}{M_\odot} \right)^{-2.5} \text{yr} \, . \qquad \text{(B.6)}$$

Using this argument, we observe that stars of higher mass conclude their lives on the main sequence much faster than stars of lower mass. The Sun will remain on the main sequence for about eight to ten billion years, with about half of this time being over already. In comparison, very luminous stars, like O and B stars, will have a lifetime on the main sequence of only a few million years before they have exhausted their hydrogen fuel.

In the course of their evolution on the main sequence, stars move away only slightly from the ZAMS in the HRD, towards somewhat higher luminosities and lower effective temperatures. In addition, the massive stars in particular can lose part of their initial mass by stellar winds. The evolution after the main-sequence phase depends on the stellar mass. Stars of very low mass, $M \lesssim 0.7 M_\odot$, have a lifetime on the main sequence which is longer than the age of the Universe, therefore they cannot have moved away from the main sequence yet.

For massive stars, $M \gtrsim 2.5 M_\odot$, central hydrogen burning is first followed by a relatively brief phase in which the fusion of hydrogen into helium takes place in a shell outside the center of the star. During this phase, the star quickly moves to the "right" in the HRD, towards lower temperatures, and thereby expands strongly. After this phase, the density and temperature in the center rise so much as to ignite the fusion of helium into carbon. A central helium-burning zone will then establish itself, in addition to the source in the shell where hydrogen is burned. As soon as the helium in the core has been exhausted, a second shell source will form fusing helium. In this stage, the star will become a red giant or supergiant, ejecting part of its mass into the ISM in the form of stellar winds. Its subsequent evolutionary path depends on this mass loss. A star with an initial mass $M \lesssim 8 M_\odot$ will evolve into a white dwarf, which will be discussed further below.

For stars with initial mass $M \lesssim 2.5 M_\odot$, the helium burning in the core occurs explosively, in a so-called helium flash. A large fraction of the stellar mass is ejected in the course of this flash, after which a new stable equilibrium configuration is established, with a helium shell source burning beside the hydrogen-burning shell. Expanding its radius, the star will evolve into a red giant or supergiant and move along the asymptotic giant branch (AGB) in the HRD.

The configuration in the helium shell source is unstable, so that its burning will occur in the form of pulses. After some time, this will lead to the ejection of the outer envelope which then becomes visible as a *planetary nebula*. The remaining central star moves to the left in the HRD, i.e., its temperature rises considerably (to more than 10^5 K). Finally, its radius gets smaller by several orders of magnitude, so that the the stars move downwards in the HRD, thereby slightly reducing its temperature: a white dwarf is born, with a mass of about $0.6 M_\odot$ and a radius roughly corresponding to that of the Earth.

If the initial mass of the star is $\gtrsim 8 \, M_\odot$, the temperature and density at its center become so large that carbon can also be fused. Subsequent stellar evolution towards a core-collapse supernova is described in Sect. 2.3.2.

The individual phases of stellar evolution have very different time-scales. As a consequence, stars pass through certain regions in the HRD very quickly, and for this reason stars at those evolutionary stages are never or only rarely found in the HRD. By contrast, long-lasting evolutionary stages like the main sequence or the red giant branch exist, with those regions in an observed HRD being populated by numerous stars.

C. Units and Constants

In this book, we consistently used, besides astronomical units, the Gaussian cgs system of units, with lengths measured in cm, masses in g, and energies in erg. This is the commonly used system of units in astronomy. In these units, the speed of light is $c = 2.998 \times 10^{10}\,\mathrm{cm\,s^{-1}}$, the masses of protons, neutrons, and electrons are $m_\mathrm{p} = 1.673 \times 10^{-24}$ g, $m_\mathrm{n} = 1.675 \times 10^{-24}$ g, and $m_\mathrm{e} = 9.109 \times 10^{-28}$ g, respectively.

Frequently used units of length in astronomy include the Astronomical Unit, thus the average separation between the Earth and the Sun, where $1\,\mathrm{AU} = 1.496 \times 10^{13}$ cm, and the parsec (see Sect. 2.2.1 for the definition), $1\,\mathrm{pc} = 3.086 \times 10^{18}$ cm. A year has $1\,\mathrm{yr} = 3.156 \times 10^{7}$ s. In addition, masses are typically specified in Solar masses, $1 M_\odot = 1.989 \times 10^{33}$ g, and the bolometric luminosity of the Sun is $L_\odot = 3.846 \times 10^{33}\,\mathrm{erg\,s^{-1}}$.

In cgs units, the value of the elementary charge is $e = 4.803 \times 10^{-10}\,\mathrm{cm^{3/2}\,g^{1/2}\,s^{-1}}$, and the unit of the magnetic field strength is one Gauss, where $1\,\mathrm{G} = 1\,\mathrm{g^{1/2}\,cm^{-1/2}\,s^{-1}} = 1\,\mathrm{erg^{1/2}\,cm^{-3/2}}$. One of the very convenient properties of cgs units is that the energy density of the magnetic field in these units is given by $\rho_B = B^2/(8\pi)$ – the reader may check that the units of this equation is consistent.

X-ray astronomers measure energies in electron Volts, where $1\,\mathrm{eV} = 1.602 \times 10^{12}$ erg. Temperatures can also be measured in units of energy, because $k_\mathrm{B} T$ has the dimension of energy. They are related according to $1\,\mathrm{eV} = 1.161 \times 10^{4}\,k_\mathrm{B}\,\mathrm{K}$. Since we always use the Boltzmann constant k_B in combination with a temperature, its actual value is never needed. The same holds for Newton's constant of gravity which is always used in combination with a mass. Here one has $G\,M_\odot\,c^{-2} = 1.495 \times 10^{5}$ cm.

The frequency of a photon is linked to its energy according to $h_\mathrm{P}\nu = E$, and we have the relation $1\,\mathrm{eV}\,h_\mathrm{P}^{-1} = 2.418 \times 10^{14}\,\mathrm{s^{-1}} = 2.418 \times 10^{14}$ Hz. Accordingly, we can write the wavelength $\lambda = c/\nu = h_\mathrm{P} c/E$ in the form

$$\frac{h_\mathrm{P} c}{1\,\mathrm{eV}} = 1.2400 \times 10^{-4}\,\mathrm{cm} = 12\,400\,\text{Å}\,.$$

Peter Schneider, Units and Constants.
In: Peter Schneider, Extragalactic Astronomy and Cosmology. pp. 431–431 (2006)
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