

ASTRO 301 (50405) : HOMEWORK 4

Assigned on Tue Nov 21. Due on Th Nov 28 in class

Instructions

- The number of points for each question is indicated in brackets. **In order to get full credit, you must show the method that you used to derive the answer.** See the class website (<http://www.as.utexas.edu/~sj/a301-fa06/>) for the grading policy.
- See Appendix A of your textbook for the value of constants. A few are listed here:
1 pc $\sim 3 \times 10^{16}$ m; 1 $M_{\odot} \sim 2 \times 10^{30}$ kg;
Gravitational constant $G \sim 6.7 \times 10^{-11}$ N m² kg⁻²;
Planck's constant $h \sim 6.6 \times 10^{-34}$ J s; Speed of light $c \sim 3 \times 10^8$ m s⁻¹

1a. Describe why a thin layer of gas of a certain composition can produce emission lines only at certain specific wavelengths. [10 pts]

1b. When a hydrogen atom is in its ground (un-excited) state, its single electron lies in the lowest allowed energy level. In order to excite its electron to the next allowed energy level, which is 1.6×10^{-18} J higher, what is the wavelength of the photon that the hydrogen atom must absorb? [10 pts]

2. In this question, you will determine the amount of dark matter in a spiral galaxy. In the disk of such a galaxy, astronomers have mapped the circular speed v of gas clouds, which are orbiting about the galaxy center on circular orbits at different radii R . As discussed in class, the gas cloud at a radius R moves on a circular orbit because of the force of gravity that is exerted on it by the total mass M of the galaxy enclosed inside the radius R . We can further show that the circular speed v of the gas cloud depends on R and M as follows:

$$v^2 = \frac{GM}{R} \quad (1)$$

where v is in m s⁻¹, R is in m, M is in kg, and the Gravitational constant G is 6.67×10^{-11} N m² kg⁻².

2a. For a gas cloud at a radius $R \sim 8.5$ kiloparsec, the circular speed v is measured to be 2×10^5 m s⁻¹. Calculate the mass M of the galaxy inside this radius in kg. [10 pts]

2b. Express M in solar masses. [3 pts]

2c. The galaxy mass M enclosed inside the radius R is made up of stars, dust, gas, and dark matter. Optical and radio observations show that inside the radius R , the mass of stars is $1.3 \times 10^{10} M_{\odot}$, while the mass of gas and dust amounts to $1 \times 10^9 M_{\odot}$. Estimate the amount of dark matter inside the radius R , expressing your answer in M_{\odot} . [10 pts].

2d. What fraction of the total mass M of the galaxy inside the radius R is made up of dark matter? [7 pts].

3. For this question, read the article "Hubble's Really Big Picture" posted on the class website. Then, use the article and the concepts in the lectures to answer the questions below.

3a. An observational survey of galaxies can yield useful insight on the evolution of galaxies if satisfies four criteria: it covers a large area of the sky; it has a large depth or exposure time; it

has images from the Hubble Space Telescope; and it has spectra yielding redshifts for the galaxies mapped. Describe why each of the four criteria is needed [**4x5 = 20 pts**].

3b. Describe three aspects of the evolution of spiral (disk) galaxies that GEMS can help us to learn about. [**3x5 = 15 pts**].

END OF ASSIGNMENT