

# “XRbinary”: A Program to Calculate the Orbital Light Curves of X-Ray Binary Stars <sup>1</sup>

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## I. Introduction

Program XRbinary calculates light curves of low-mass X-ray binary stars. The model used by the program has two stars in circular orbits around their center of mass. The stars obey Kepler’s law:

$$G(m_1 + m_2) = \omega^2 a_s^3 \quad (1)$$

$$\omega = \frac{2\pi}{P} \quad (2)$$

where  $G$  is the gravitational constant,  $m_1$  and  $m_2$  are the masses of the two stars,  $\omega$  is the orbital angular velocity of the binary,  $a_s$  is the separation of the centers of mass of the two stars, and  $P$  is the orbital period. Star 1, the “primary star,” is surrounded by an accretion disk. Star 2 is called the “secondary star.”

Because the program is optimized for calculating the light curves of X-ray binary stars, it has several distinctive features:

- The secondary star is assumed to fill its Roche lobe.
- The primary star is much smaller than any other structure in the system.
- Irradiation is important and can dominate local sources of heat.
- The geometry of the accretion disk can be complicated.
- The emitting material may not be confined to the stars and accretion disk. For example there may be an accretion disk corona or an advective flow region.
- Because the geometry can be complex, the program uses ray-tracing to calculate light curves and irradiation.

## II. The Properties of the Primary Star (= Star 1)

Star 1 is a small sphere if it represents neutron star, or an invisible point mass if it represents a black hole. In either case, it is much smaller than all other dimensions in the system and is unresolved by any of the grids used in calculating the light curves.

The primary star has luminosity  $L_1$  and effective temperature  $T_1$ . The only difference between a neutron star and a black hole is that  $L_1 = 0$  for a black hole. The flux at a distance  $d$  from the primary is

$$F_1 = \frac{L_1}{4\pi d^2}. \quad (3)$$

The primary emits black body radiation. The specific intensity emitted by a black body is independent of angle and, therefore, the relation between specific intensity and flux is:

$$F_1 = \int_0^{\pi/2} I_1 \cos \theta \sin \theta d\theta d\phi = \pi I_1 \quad (4)$$

Furthermore, the flux from a spherical star with radius  $R_1$  seen by a distant observer is

$$\mathcal{F}_1(\lambda) = \int_0^{\pi/2} I_1(\lambda) R_1^2 \cos \theta \sin \theta d\theta d\phi = \pi R_1^2 I_1(\lambda). \quad (5)$$

Making use of the definition of effective temperature  $L_1 = 4\pi R_1^2 \sigma T_1^4$ , we find

$$\mathcal{F}_1(\lambda) = \frac{L_1}{4\sigma T_1^4} I(\lambda) = \frac{L_1}{4\pi\sigma T_1^4} B(\lambda) \quad (6)$$

where  $B(\lambda)$  is the black body flux:

$$B(\lambda) = \frac{2\pi h c^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1} \quad (7)$$

Equation 3 can be used to calculate irradiation and heating by star 1 and equation 6 can be used to calculate the direct contribution of flux from star 1 to the light curve.

### III. The Geometry and Temperature of the Secondary Star (= Star 2)

#### III.1. Geometry

The surface of the secondary star is assumed to coincide with its largest confining zero-velocity surface, its Roche lobe. The zero velocity surfaces are the surfaces that satisfy the equation (See Appendix B)

$$V = \frac{Gm_1}{a_s} \left[ -\frac{a_s}{r_1} - \frac{q a_s}{r_2} - \frac{1}{2}(1+q) \left( \frac{\rho_{\text{cm}}}{a_s} \right)^2 \right] = \text{constant} \quad (8)$$

where  $q = m_2/m_1$  is the mass ratio,  $r_1$  and  $r_2$  are the distances of a point from the centers of the stars, and  $\rho_{\text{cm}}$  is the perpendicular distance of the point from the rotation axis of the binary system. The center of mass of the binary is along the axis joining the centers of the stars at a distance  $r_{\text{cm}}$  from the center of the secondary star, where

$$r_{\text{cm}} = \frac{m_1}{m_1 + m_2} a_s = \frac{1}{1+q} a_s. \quad (9)$$

Since the surface of the star is coincident with a zero-velocity surface, the unit vector normal to its surface is given by

$$\hat{n} = \frac{\nabla V}{|\nabla V|} \quad (10)$$

where  $\nabla$  is the gradient operator and  $\nabla V$  is evaluated at the star's surface. The explicit form of  $\nabla V$  can be messy (see Appendix C) and, except for a few special cases, is more easily calculated numerically. The surface gravity of the star is

$$\vec{g} = -\nabla V, \quad (11)$$

where, again,  $\nabla V$  is evaluated at the star's surface.

The Lagrangian points are the five places where  $\nabla V = 0$ ; the inner Lagrangian point is the Lagrangian point along the line joining the centers of the two stars. The Roche lobes are the surfaces around the two stars on which  $V$  equals its value at the inner Lagrangian point. Equation 10 is undefined at the inner Lagrangian point, but  $\hat{n} = \pm\hat{r}$  everywhere along the line joining the centers of the stars, where  $\hat{r}$  is the unit vector in the direction from the center of star 2 (the sign switches at the inner Lagrangian point). We set  $\hat{n} = \hat{r}$  at the inner Lagrangian point also.

The vector element of surface area is

$$d\vec{A} = \frac{\hat{n} dS}{\hat{n} \cdot \hat{r}} \quad (12)$$

where  $dS$  is an the scalar element of area on the surface of a sphere centered on the origin. The factor  $1/(\hat{n} \cdot \hat{r})$  is a projection factor introduced because the surface of the star is not perpendicular to  $\hat{r}$ . The expression for  $dS$  depends on the coordinate system.

The vector element of surface area projects to an area

$$\hat{n}_o \cdot d\vec{A} \quad (13)$$

on the sky, where  $\hat{n}_o$  is the unit vector in the direction of the observer. The total visible projected surface area is then

$$A_{sky} = \int \hat{n}_o \cdot d\vec{A} = \int \int (\hat{n}_o \cdot \hat{n}) \frac{dS}{\hat{n} \cdot \hat{r}} \quad (14)$$

where the integral is taken over the visible surface of the star. A surface element of an isolated star is visible if its surface normal satisfies the condition

$$\hat{n}_o \cdot d\vec{A} > 0 \quad (15)$$

or, equivalently,  $\hat{n}_o \cdot \hat{n} > 0$ . If other components of the system can obscure the star, determination of visibility becomes more complicated (see Section IV.2).

### III.2. Gravity Darkening and Local Effective Temperature

The local effective temperature  $T_{eff}$  of the secondary star is given by the gravity-darkening law

$$T_{eff} = \langle T_{eff} \rangle \left( \frac{|g|}{\langle |g| \rangle} \right)^\beta \quad (16)$$

where  $\langle T_{eff} \rangle$  and  $\langle |g| \rangle$  are the mean effective temperature and mean surface gravity of the star, and  $\beta$  is the gravity darkening exponent. The value of  $\langle T_{eff} \rangle$  is an input parameter

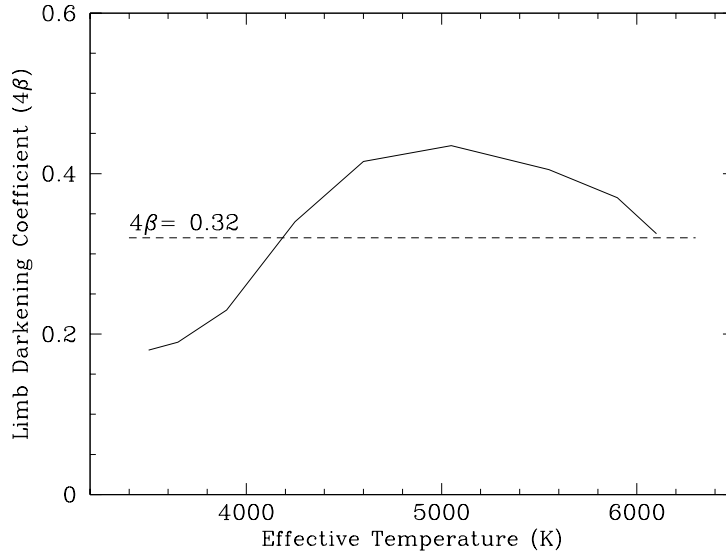


Figure 1: The gravity darkening coefficient as a function of effective temperature.

but  $\langle |g| \rangle$  is calculated by the program. To provide a self consistent way to calculate  $\langle |g| \rangle$ , we define the mean effective temperature to be

$$L_2 = \sigma \langle T_{eff} \rangle^4 S = \int \sigma T_{eff}^4 dS = \int \sigma \langle T_{eff} \rangle^4 \left( \frac{|g|}{\langle |g| \rangle} \right)^{4\beta} dS \quad (17)$$

where  $L_2$  is the luminosity of star 2,  $S$  is its total surface area, and the integral is taken over the entire surface of the star. We have, then,

$$\langle |g| \rangle^{4\beta} = \frac{1}{S} \int |g|^{4\beta} dS \quad (18)$$

The gravity darkening coefficient is equal to 0.25 for stars with radiative envelopes and lies between  $\sim 0.05$  and  $\sim 0.11$  for stars with convective envelopes. The gravity darkening coefficient used in the program depends only on temperature and is an approximation to the coefficients for main sequence stars given by Claret (2000, A& A, 359, 289). The dependence is shown in Figure 1.

The secondary star can have (roughly) circular spots that are hotter or colder than the temperature given by equation 16. Each spot is defined by the position of its center in  $\theta$  and  $\phi$ , by its angular radius as seen from the center of the secondary star, and by its temperature, which is specified by the ratio  $T_{spot}/T_{eff}$ . Note that specifying the temperature this way means that the temperature varies within a spot as  $T_{eff}$  varies. If there is more than one spot and if the spots overlap, the fraction by which the temperatures in the overlap regions are reduced is the product of the factors for the individual overlapping spots.

If the user asks the program to included the effects of irradiation, the local effective temperatures across the surface of the secondary star can be greatly increased (see Section VI).

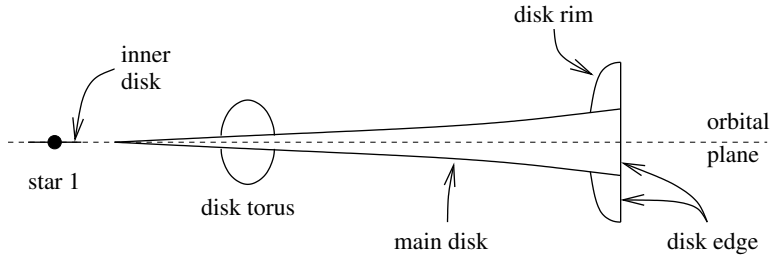


Figure 2: The components of the accretion disk.

#### IV. The Geometry and Temperature of the Accretion Disk

In the current version of the program the accretion disk is symmetric about the orbital plane. It consists of an elliptical “main disk” plus a “disk rim,” a “disk edge,” a “disk torus,” and “inner disk” (see Figure 2). The height of the disk above the orbital plane is the sum of the heights of the individual components:

$$H = H_{main} + H_{rim} + H_{torus}, \quad (19)$$

although the rim and torus are not allowed to overlap.

The projection of the accretion disk onto the orbital plan is elliptical with star 1 at one focus. The properties of the disk are specified in an auxiliary cylindrical coordinate system  $(a, \zeta, h)$  in which the disk is circular and are then mapped to the  $(\rho, \zeta, h)$  coordinate system

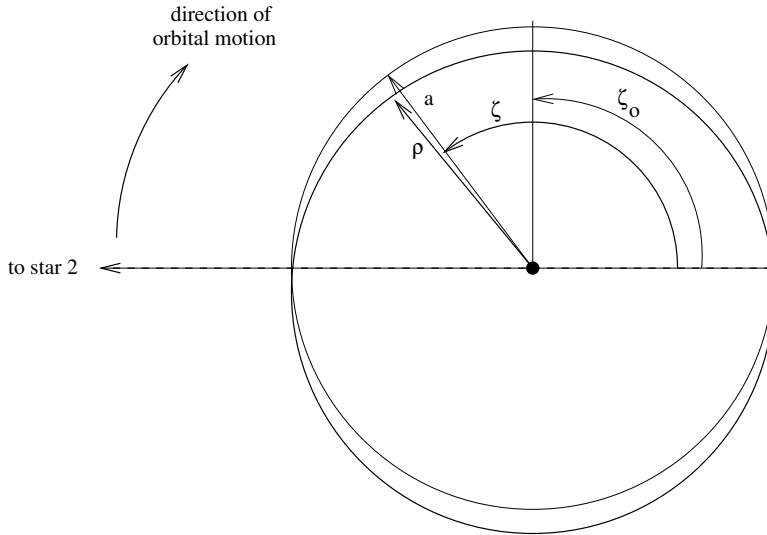


Figure 3: Top view of the eccentric disk. All disk properties are specified as if the disk were circular with radial coordinate  $a$ . The disk is then “stretched” to make it elliptical by means of the transformation  $\rho = a(1 - e^2)/[1 + e \cos(\zeta - \zeta_0)]$ , where  $\zeta_0$  is the longitude of periastron.

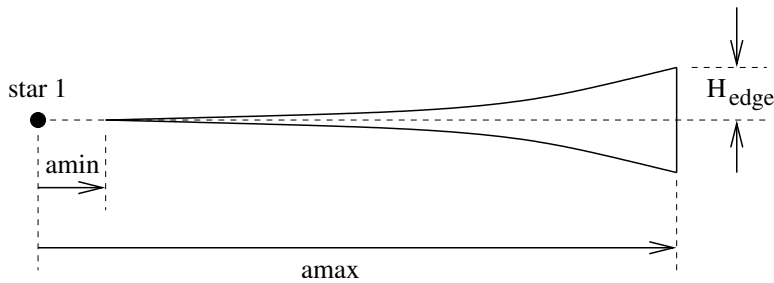


Figure 4: Side view of the main disk, defining the meaning of its parameters.

by the transformation

$$\rho = \frac{a(1 - e^2)}{1 + e \cos(\zeta - \zeta_0)} \quad (20)$$

where  $e$  is the eccentricity of the disk and  $\zeta_0$  is the angle at which the disk is closest to star 1 (see Figure 3). Thus, the disk has a minimum radius  $a_{min}$  and maximum radius  $a_{max}$ , which map to

$$\rho_{min} = \frac{a_{min}(1 - e^2)}{1 + e \cos(\zeta - \zeta_0)} \quad (21)$$

$$\rho_{max} = \frac{a_{max}(1 - e^2)}{1 + e \cos(\zeta - \zeta_0)} \quad (22)$$

In cylindrical coordinates the vector element of surface area on the top and bottom surfaces of the disk is

$$d\vec{A} = \frac{\hat{n} \rho d\rho d\zeta}{\hat{n} \cdot \hat{h}}, \quad (23)$$

where here  $\hat{n}$  is the unit vector normal to the surface of the disk and  $\hat{h}$  is the unit vector in the direction perpendicular to the orbital plane (positive  $h$  is antiparallel to  $\vec{\omega}$ ). The vector element of surface area on the edge of the disk is

$$d\vec{A} = \frac{\hat{n} \rho d\zeta dh}{\hat{n} \cdot \hat{\rho}}, \quad (24)$$

where  $\hat{\rho}$  is the unit vector in the  $\rho$  direction. The unit vectors normal to the surface of the disk are calculated numerically (see Appendix A for additional discussion).

**The Main Disk:** The height of the main disk is a power law in  $a$ :

$$H_{main} = H_{edge} \left( \frac{a - a_{min}}{a_{max} - a_{min}} \right)^{\beta_H}, \quad (25)$$

where  $H_{edge}$  is the height of the main disk at its outer edge (see Figure 4). Note that the disk tapers to  $H_{main} = 0$  at its inner edge, so its inner edge does not have any vertical extent.

The dependence of the temperature on radius can be either an arbitrary power law or the distribution corresponding to a steady-state, optically-thick, viscous disk. The power law is

$$T_{main} = \begin{cases} Ka^{\beta_T}, & a_{min} \leq a \leq a_{max} \\ 0, & \text{otherwise} \end{cases}. \quad (26)$$

The normalization constant  $K$  is set so that the temperature distribution gives the desired total luminosity of the main disk. It is calculated from

$$L_{main} = 2 \int_{a_{min}}^{a_{max}} \sigma T_{main}^4 2\pi r dr, \quad (27)$$

where  $L_{main}$  is the desired luminosity. The temperature distribution for a steady-state viscous disk is given by

$$T_{main}^4 = \begin{cases} \frac{K}{a^3} \left[ 1 - \left( \frac{a_{min}}{a} \right)^{1/2} \right], & a_{min} \leq a \leq a_{max} \\ 0, & \text{otherwise} \end{cases}, \quad (28)$$

where, once again, the normalization constant  $K$  is set to give the desired total disk luminosity through equation 27.

The main disk may also have localized spots that are hotter or colder than the temperature given by equation 26. The properties of each spot are specified by its boundaries in  $\zeta$  and  $a$ : ( $\zeta_{min}$ ,  $\zeta_{max}$ ,  $a_{min}$ ,  $a_{max}$ ); and by its temperature, which is specified by the ratio  $T_{spot}/T_{main}$ . Note that specifying the temperature this way means that the temperature varies from point to point within a spot as  $T_{main}$  varies. If there is more than one spot and if the spots overlap, the fraction by which the temperature in the overlap regions are reduced is the product of the factors for the individual overlapping spots.

**The Disk Edge:** The outer edge of the disk is everywhere perpendicular to the orbital plane. Its height is equal to the sum of the heights of the main disk and the disk rim at  $a_{max}$ . The edge has the same temperature everywhere except for a rectangular spot with uniform temperature that extends from the bottom of the disk edge to its top. The position of the spot is specified by  $\zeta_{mid}$  and  $\zeta_{width}$ , the location of the center of the spot and its full width. If the temperature of the spot is less than the temperature of the rest of the edge, the program ignores the spot.

**The Disk Rim:** The cross section of the of the disk rim is a quarter ellipse in  $a$ :

$$H_{rim} = H_{rim}(\zeta) \left[ 1.0 - \left( \frac{a_{max} - a}{a_{width}} \right)^2 \right]^{1/2} \quad (29)$$

where  $H_{rim}(\zeta)$  is the height of the rim at  $a = a_{max}$  and  $H_{rim} = 0$  outside the range  $a_{max} \geq a \geq (a_{max} - a_{width})$ . Note that the rim is flat topped since  $dH_{rim}/da = dH_{rim}/d\rho = 0$  at  $a = a_{max}$ , but  $H_{rim} + H_{main}$  is generally not flat topped. The height of the rim can vary



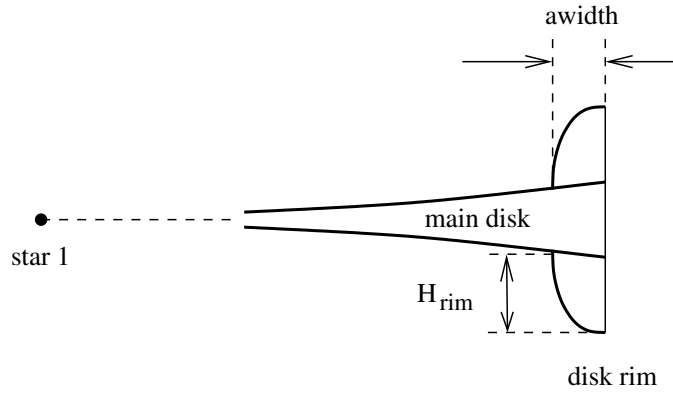


Figure 5: Side view of the disk and disk rim, defining the meaning of the rim parameters.

around the disk by making  $H_{rim}(\zeta)$  vary with  $\zeta$ .  $H_{rim}(\zeta)$  can be specified in either of two ways. It can be a sinusoidal function of  $\zeta$ :

$$H_{rim}(\zeta) = \frac{1}{2}(H_{max} + H_{min}) + \frac{1}{2}(H_{max} - H_{min}) \cos(\zeta - \zeta_{max}) \quad (30)$$

or it can be specified by its value at a series of points around the rim:

$$\begin{array}{ll} \zeta_1 & H_{rim}(\zeta_1) \\ \zeta_2 & H_{rim}(\zeta_2) \\ \vdots & \vdots \\ \zeta_n & H_{rim}(\zeta_n) \end{array}$$

The value of  $H_{rim}(\zeta)$  between the points is calculated by linear interpolation.

The outer side of the disk rim is vertical and takes on the same temperature as the edge of the main disk. In other words, it is handled as an extension of the disk edge.

The temperature on the inner side of the rim is a function of only  $\zeta$ . The temperature can be specified in either of two ways. It can be a sinusoidal function of  $\zeta$ :

$$T_{rim}(\zeta) = \frac{1}{2}(T_{max} + T_{min}) + \frac{1}{2}(T_{max} - T_{min}) \cos(\zeta - \zeta_{max}) \quad (31)$$

or it can be specified by its value at a series of points around the rim:

$$\begin{array}{ll} \zeta_1 & T_{rim}(\zeta_1) \\ \zeta_2 & T_{rim}(\zeta_2) \\ \vdots & \vdots \\ \zeta_n & T_{rim}(\zeta_n) \end{array}$$

The value of  $T_{rim}(\zeta)$  between the points is calculated by linear interpolation. If the temperature of the rim at any  $\zeta$  drops below the temperature of the underlying main disk at that

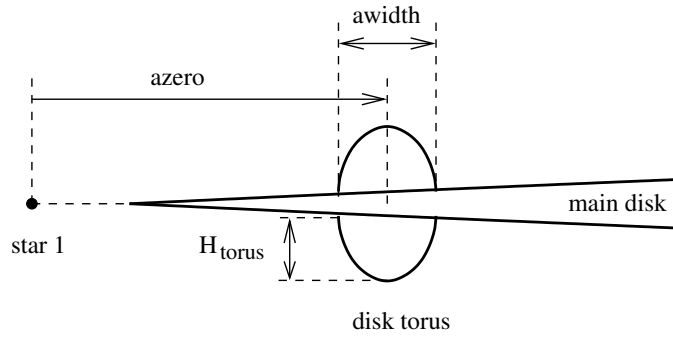


Figure 6: Side view of the disk and disk torus, defining the meaning of the torus parameters.

point, the temperature of the rim there is set to the temperature of the underlying main disk.

**The Disk Torus:** The cross section of the of the disk torus is a half ellipse in  $a$ :

$$H_{torus} = H_{torus}(\zeta) \left[ 1.0 - \left( \frac{a_{zero} - a}{a_{width}/2} \right)^2 \right]^{1/2} \quad (32)$$

where  $H_{torus}(\zeta)$  is the height of the rim at  $a = a_{zero}$  and  $H_{torus} = 0$  for  $|a_{zero} - a| > a_{width}/2$ . The height of the torus can vary around the disk by making  $H_{torus}(\zeta)$  vary with  $\zeta$ .  $H_{torus}(\zeta)$  can be specified in either of two ways. It can be a sinusoidal function of  $\zeta$ :

$$H_{torus}(\zeta) = \frac{1}{2}(H_{max} + H_{min}) + \frac{1}{2}(H_{max} - H_{min}) \cos(\zeta - \zeta_{max}) \quad (33)$$

(the values of  $H_{min}$ ,  $H_{max}$ , and  $\zeta_{max}$  here will generally be different from their values in equation 30), or it can be specified by its value at a series of points around the rim:

$$\begin{array}{ll} \zeta_1 & H_{torus}(\zeta_1) \\ \zeta_2 & H_{torus}(\zeta_2) \\ \vdots & \vdots \\ \zeta_n & H_{torus}(\zeta_n) \end{array}$$

The value of  $H_{torus}(\zeta)$  between the points is calculated by linear interpolation.

The temperature on the torus is a function of only  $\zeta$ . Its temperature can be specified in either of two ways. It can be a sinusoidal function of  $\zeta$ :

$$T_{torus}(\zeta) = \frac{1}{2}(T_{max} + T_{min}) + \frac{1}{2}(T_{max} - T_{min}) \cos(\zeta - \zeta_{max}) \quad (34)$$

(again, the values of  $T_{min}$ ,  $T_{max}$ , and  $\zeta_{max}$  here will generally not be the same as their values in equation 31) or it can be specified by its value at a series of points around the rim:

$$\begin{array}{ll}
\zeta_1 & T_{torus}(\zeta_1) \\
\zeta_2 & T_{torus}(\zeta_2) \\
\vdots & \vdots \\
\zeta_n & T_{torus}(\zeta_n)
\end{array}$$

The value of  $T_{torus}(\zeta)$  between the points is calculated by linear interpolation. If the temperature of the torus at any point drops below the temperature of the underlying main disk at that point, the temperature of the torus there is set to the temperature of the underlying main disk.

**The Inner Disk:** The primary reason for including the inner disk is to allow irradiation of the outer disk and the secondary star from a hot source at the center of the disk when the compact star is a black hole. The inner disk differs from a star (or any other quasi-spherical source) because it is flat, has zero thickness, and lies entirely in the orbital plane.

The flux from the inner disk is handled in much the same way as the flux from the primary star except for the different geometry. The flux at a distance  $d$  from the inner disk is

$$F_{id} = \frac{L_{id}}{2\pi d^2} \cos \theta. \quad (35)$$

The flux from an inner disk with radius  $R_{id}$  seen by a distant observer is

$$\mathcal{F}(\lambda) = \pi R_{id}^2 I(\lambda) \cos \theta. \quad (36)$$

Making use of the definition of effective temperature  $L_{id} = 2\pi R_{id}^2 \sigma T_{id}^4$ , we find

$$\mathcal{F}_{id}(\lambda) = \frac{L_{id}}{2\sigma T_{id}^4} I(\lambda) \cos \theta = \frac{L_{id}}{2\pi\sigma T_{id}^4} B(\lambda) \cos \theta \quad (37)$$

where  $B(\lambda)$  is given by equation 7. Equation 35 can be used to calculate irradiation and heating by the inner disk and equation 37 can be used to calculate the direct contribution of flux from the inner disk to the light curve.

## V. The Properties of the Accretion Disk Corona

The accretion disk corona (ADC) model used in XRbinary Version 2.1 is extremely primitive. It allows one to calculate approximate orbital lightcurves at ultraviolet to infrared wavelengths that include the effects of heating by the ADC. But there is no direct flux from the ADC at any wavelength, including X-ray wavelengths – this must await the next version of the program. Furthermore, the ADC is approximated by two point sources of flux, one directly above and one directly below star 1, each emitting half the total flux from the ADC.

The ADC has luminosity  $L_{ADC}$  and the two points are located at  $x = 0$ ,  $y = \pm h_{adc}$ ,  $z = a$ . The flux at a distance  $d$  from one point is

$$F_{ADC} = \frac{1}{2} \frac{L_{ADC}}{4\pi d^2}. \quad (38)$$

The choice of  $h_{ADC}$  is open to the user. One possibility is to place the points at the “centers of gravity” of the emission from the upper and lower halves of the ADC. With this choice a cylindrical ADC extending from  $y = -H$  to  $H$  with uniform emission would be represented by two points at  $y = \pm H/2$ .

## VI. The Orbital Light Curves

### VI.1 The Basic Calculation

The observed wavelength-dependent flux from the system is given by

$$\mathcal{F}(\lambda) = \int I(T_{eff}, g, \mu, \lambda) \hat{n}_o \cdot d\vec{A} \quad (39)$$

where the integral is taken over all visible surfaces in the system, and

$$\begin{aligned} I &= \text{the local specific intensity} \\ T_{eff} &= \text{local effective temperature} \\ g &= \text{local gravity} \\ \mu &= \hat{n}_o \cdot \hat{n} \\ \lambda &= \text{wavelength} \end{aligned}$$

The light curve is generally measured in a bandpass X (a “filter”) characterized by a normalized response function

$$R_X(\lambda) = S_X(\lambda) / \int S_X(\lambda) d\lambda, \quad (40)$$

where  $S_X$  is the unnormalized response function. The observed flux density is, then,

$$\mathcal{F}_X = \int R_X(\lambda) \mathcal{F}(\lambda) d\lambda. \quad (41)$$

$\mathcal{F}_X$  is a function of orbital phase; the light curve is a plot of  $\mathcal{F}_X$  against orbital phase.

For the most accurate calculations, equation 41 needs a large grid of specific intensities as a function of  $T_{eff}$ ,  $\log g$ ,  $\mu$ , and  $\lambda$ . The calculations can be simplified by assuming that the angular dependence can be separated into a limb-darkening function, and that the limb-darkening coefficients are constant and equal to their values at a wavelength,  $\lambda_X$ . The specific intensity becomes, then:

$$I = I_{\perp}(T_{eff}, g, \lambda) H(T_{eff}, g, \mu, \lambda_X) \quad (42)$$

where  $H(T_{eff}, g, \mu, \lambda_X)$  is the limb darkening function and  $I_{\perp}(T_{eff}, g, \lambda)$  is the specific intensity emitted perpendicular to the surface

$$I_{\perp}(T_{eff}, g, \lambda) = I(T_{eff}, g, \mu = 1, \lambda). \quad (43)$$

Equation 41 now simplifies to

$$\mathcal{F}_X = \int \left[ \int R_X(\lambda) I_{\perp}(T_{eff}, g, \lambda) d\lambda \right] H(T_{eff}, g, \mu, \lambda_X) \hat{n}_o \cdot d\vec{A} \quad (44)$$

The integral over wavelength is independent of geometry and can, therefore, be precalculated. Defining

$$\bar{I}_{\perp X}(T_{eff}, g) = \int R_X(\lambda) I_{\perp}(T_{eff}, g, \lambda) d\lambda, \quad (45)$$

equation 44 takes the simple form

$$\mathcal{F}_X = \int \bar{I}_{\perp X}(T_{eff}, g) H(T_{eff}, g, \mu, \lambda_X) \hat{n}_o \cdot d\vec{A}, \quad (46)$$

where the intergral is taken over all visible surfaces. Equation 46 is the basic working equation for the XRbinary program.

## VI.2. The Use of Ray Tracing to Determine Visibility

The previous section contains integrals that must be evaluated over all visible surfaces in the system. To determine visibility the program traces rays emitted by the surfaces of the various components of the binary system in the direction of the sun. A surface element is deemed visible if a ray from the element escapes the binary without passing through any other surface element. Ray tracing is computationally intensive (more than 99% of the programs computational effort is spent on ray tracing) but can handle complicated geometries. For example, XRbinary correctly handles the visibility of features that are on the lower surface of the disk but are visible through the hole in the center of the disk. Pixelization causes complications for ray tracing algorithms; see comments in the code for a brief discussion.

## VI.3. The Flux from the Secondary Star

The filter sets currently incorporated into the program are the Johnson UBV, the Cousins RI, and the Johnson JHK sets. In addition, one can specify one or more sharp-edged, flat bandpasses between pairs of wavelengths (the ‘‘SQUARE’’ bandpass). The square bandpass assumes the emitting body is a black body with no limb darkening – a poor approximation to the secondary star unless it is heated by irradiation. A full discussion of  $\bar{I}_{\perp X}(T_{eff}, g)$  for the filters and square bandpasses is given in Appendix C. The program calculates the light curves in all filters and bandpasses simultaneously.

For temperatures greater than 3500 K and less than 8000 K, the program uses the Kurucz spectra available in the STScI calibration database. See Appendix C for a full discussion.

We adopt the limb darkening law proposed by Claret (2000, *Astron. & Astrophys.*, 363, 1081):

$$H(\mu) = \frac{I(\mu)}{I_{\perp}} = 1 - a_1(1 - \mu^{1/2}) - a_2(1 - \mu) - a_3(1 - \mu^{3/2}) - a_4(1 - \mu^2) \quad (47)$$

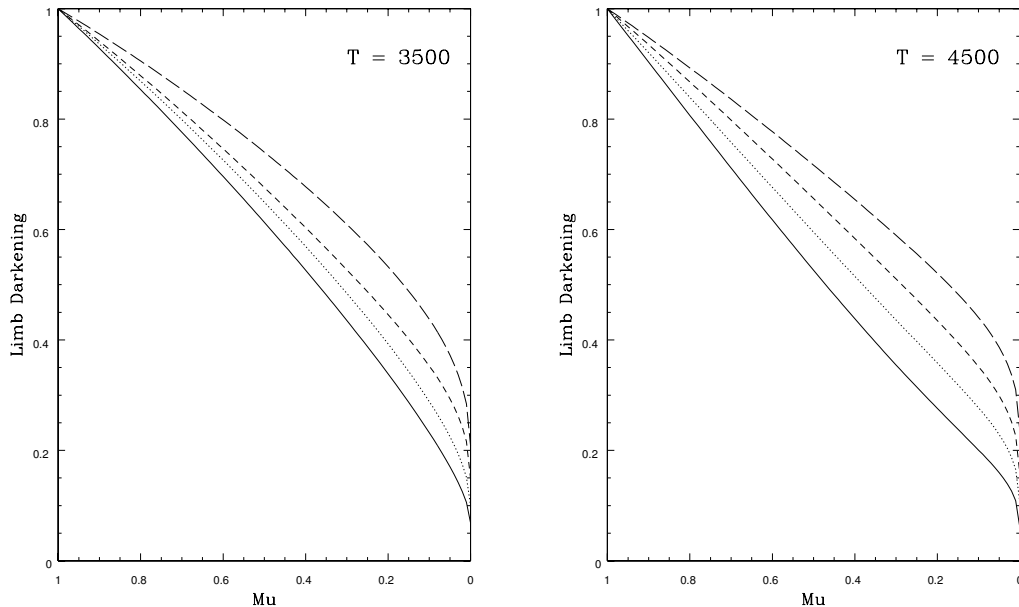


Figure 7: Examples of limb darkening from Claret (2000, *Astron. & Astrophys.*, 363, 1081). Left figure:  $\log g = 4.0$ ,  $T_{eff} = 3500$ . Right figure:  $\log g = 4.5$ ,  $T_{eff} = 4500$ . In both cases the figures correspond to the filters (from bottom to top) Stromgren v and y, and Cousins R and I.

This law does a much better job of reproducing the limb darkening near the limb than more traditional laws. The coefficients  $a_i$  have been taken from the extensive tables for various filters published by Claret in electronic form on the CDS. Figure 7 shows examples of the limb darkening.

For temperature less than 3500 K and greater than 8000 K, the program currently adopts black body radiation with no  $\mu$  dependence. The point of the upper limit is that local effective temperatures of the secondaries of low-mass X-ray binaries are not generally greater than 8000 K unless the star has been heated by irradiation. Irradiated atmospheres can depart so greatly from normal stellar atmospheres (specifically,  $T(\tau)$  can be greatly disturbed) that normal atmospheres are poor approximations to the true atmosphere and it is safer to default to black body radiation. The lower limit is set by limitations of the model atmospheres.

#### VI.4. The Flux from the Primary Star and the Accretion Disk

The program currently assumes that the flux emitted by primary star and the accretion disk has a black body distribution with no limb darkening. The program can handle black body fluxes for the same filter sets as the Kurucz atmospheres (UBVRIJHK and square bandpasses).

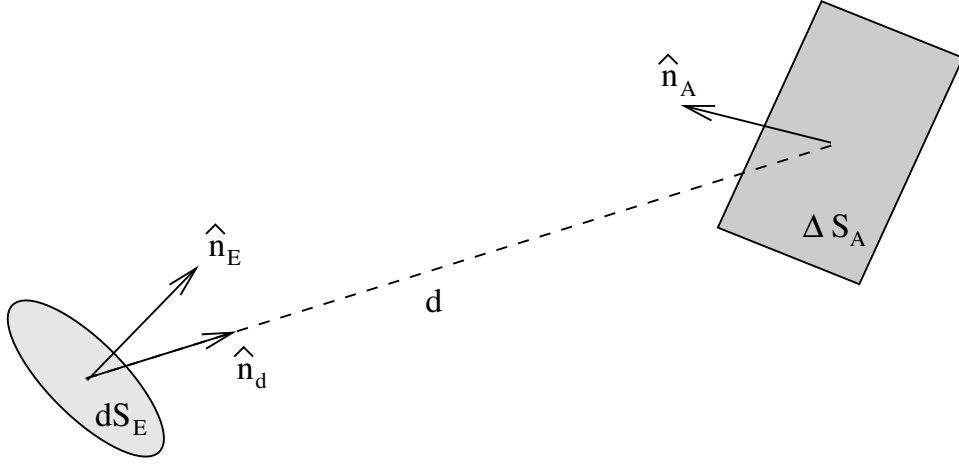


Figure 8: The geometry for calculating heating due to irradiation.

## VII. Irradiation

Let flux be emitted from an element of surface area  $d\vec{A}_E = \hat{n}_E dS_E$  and absorbed by a small surface area  $\Delta\vec{A}_A = \hat{n}_A \Delta S_A$ , where  $\hat{n}_E$  and  $\hat{n}_A$  are the unit vectors normal to the two surface areas and  $dS_E$  and  $\Delta S_A$  are the (scalar) areas. Note that  $\Delta S_A$  has a small but finite area. The distance between the centers of the two surface elements is  $d$  and the unit vector from the center of  $d\vec{A}_E$  to the center of  $\Delta\vec{A}_A$  is  $\hat{n}_d$  (see Figure 8). The flux emitted by area  $d\vec{A}_E$  towards  $\Delta\vec{A}_A$  is

$$\delta F_E = \left[ \int_{\lambda} I_E(\mu_E, \lambda) d\lambda \right] \hat{n}_d \cdot d\vec{A}_E = \left[ \int_{\lambda} I_{\perp E}(\lambda) H_E(\mu_E, \lambda) d\lambda \right] \mu_E dS_E, \quad (48)$$

where  $I_{\perp E}(\lambda)$  is the specific intensity emitted perpendicular to  $d\vec{A}_E$ ,  $H_E(\mu, \lambda)$  is the limb darkening at  $d\vec{A}_E$  and  $\mu_E = \hat{n}_d \cdot \hat{n}_E$ . The solid angle subtended by  $\Delta\vec{A}_A$  as seen from  $d\vec{A}_E$  is

$$\Delta\Omega_A = -\frac{\hat{n}_d \cdot \Delta\vec{A}_A}{d^2} = -\frac{\mu_A \Delta S_A}{d^2}, \quad (49)$$

where  $\mu_A = \hat{n}_d \cdot \hat{n}_A$ . The amount of flux intercepted by  $\Delta\vec{A}_A$  is, therefore,

$$\begin{aligned} \delta F_A &= \delta F_E \Delta\Omega_A \\ &= - \left[ \int_{\lambda} I_{\perp E}(\lambda) H_E(\mu_E, \lambda) d\lambda \right] \frac{\mu_E \mu_A}{d^2} dS_E \Delta S_A. \end{aligned} \quad (50)$$

Many surface elements will irradiate  $\Delta\vec{A}_A$ . The total flux falling on  $\Delta\vec{A}_A$  is the integral over all the emitting areas visible from  $\Delta\vec{A}_A$ :

$$\Delta F_A = \Delta S_A \int \left[ \int_{\lambda} I_{\perp E}(\lambda) H_E(\mu_E, \lambda) d\lambda \right] \frac{\mu_E \mu_A}{d^2} dS_E. \quad (51)$$

Let the effective temperature of  $\Delta S_A$  before it is heated by irradiation be  $T_{0A}$ , so that the flux it emits before heating is  $F_A = \sigma T_{0A}^4 \Delta S_A$ . If the fraction of the intercepted flux

that is thermalized and re-emitted is the albedo  $\alpha$ , the effective temperature of the surface becomes

$$\begin{aligned}\Delta S_A \sigma T_A^4 &= F_A + \alpha \Delta F_A \\ T_A^4 &= T_{0A}^4 + \frac{\alpha}{\sigma} \int \left[ \int_{\lambda} I_{\perp E}(\lambda) H_E(\mu_E, \lambda) d\lambda \right] \frac{\mu_E \mu_A}{d^2} dS_E.\end{aligned}\quad (52)$$

Since the heat capacity of the disk is low and advection is usually not important, it is likely that most of the energy absorbed by the disk is reradiated where it is absorbed. This implies  $\alpha = 1.0$  for the disk. One typically adopts  $\alpha = 0.5$  for the secondary but this value has little justification. If the secondary is heavily irradiated for a long period of time, an value of  $\alpha$  closer to 1 may be more appropriate. In any case, XRbinary requires the user to provide values of alpha for the disk and secondary star.

Under some conditions equation 52 can be much simplified. If the emitting surface is a black body – which is true for all parts of the disk and inner disk, the emitted flux reduces to

$$\int_{\lambda} I_{\perp E}(\lambda) H_E(\mu_E, \lambda) d\lambda = \frac{\sigma}{\pi} T_E^4 \quad (53)$$

and equation 52 becomes

$$T_A^4 = T_{0A}^4 + \frac{\alpha}{\pi} \int \frac{\mu_E \mu_A}{d^2} T_E^4 dS_E. \quad (54)$$

If the emitting body is a neutron star with luminosity  $L_1$ , equation 51 is replaced by

$$\Delta F_A = L_1 \frac{\Delta \Omega_A}{4\pi} = L_1 \frac{\mu_A}{4\pi d^2} \Delta S_A \quad (55)$$

and equation 52 is replaced by

$$T_A^4 = T_{0A}^4 + \frac{\alpha}{\sigma} \frac{\mu_A}{4\pi d^2} L_1. \quad (56)$$

Equations 55 and 56  $L_1$  replaced by  $L_{ADC}/2$  are also appropriate for irradiation by the ADC in the simple point-approximation model.

The angular and wavelength dependence of the radiation emitted from a surface heated by irradiation cannot be determined without detailed radiative transfer models. The current version of the program assumes that the irradiated disk, like the un-irradiated disk, emits like a black body. The heated secondary star is assumed to emit like a normal stellar atmosphere where its temperature is less than 8000 K and like a black body where its temperature is greater than 8000 K.

The integrals in equations 51 and 52 must be evaluated over all the emitting areas visible from  $\Delta \vec{A}_A$ . The program uses ray tracing to determine visibility and, again, this is extremely computationally intensive.



The XRbinary program currently allows irradiative heating of the outer accretion disk by the primary star, the inner disk and the ADC; and heating of the secondary star by the primary star, and both the inner and outer accretion disk, and the ADC. The outer disk does not heat itself.

# Appendix A. The Coordinate Systems Used by XRbinary

## A.1. The Three Coordinate Systems

XRbinary uses three different coordinate systems:

1. A Cartesian coordinate system centered on the secondary star.
2. A spherical polar coordinate system centered on the secondary star.
3. A cylindrical coordinate system centered on the primary star.

All three rotate so that all components of the binary system remain at fixed positions in the coordinate systems. The Cartesian coordinate system  $(x, y, z)$  is specified by (see Figure 9):

- The origin is at the center of mass of star 2.
- The  $z$  axis points at the center of star 1.
- The  $x$  axis lies in the plane of the orbit and points in the direction of motion of star 2 in its orbit.
- The  $y$  axis is oriented so as to make a right-handed coordinate system. The vector angular velocity,  $\vec{\omega}$ , points in the  $-y$  direction.

The spherical polar coordinate system  $(r, \theta, \phi)$  is specified by:

- The origin is at the center of mass of star 2.
- The  $r$  coordinate is the distance from the origin.
- The  $\theta = 0$  direction points at the center of star 1.
- The  $\phi = 0$  direction lies in the orbital plane and points in the direction of motion of star 2 in its orbit;  $\phi$  increases in a right-handed sense about the  $\theta = 0$  direction; and  $\vec{\omega}$  points in the  $\theta = \pi/2, \phi = 3\pi/2$  direction.

A cylindrical coordinate system  $(\rho, \zeta, h)$  star is specified by (see Figure 10):

- The origin is at the center of mass of star 1.
- The  $h$  axis is anti-parallel to  $\vec{\omega}$ .
- The  $\rho$  coordinate is the perpendicular distance from the  $h$  axis.
- The  $\zeta = 0$  direction points away from star 2;  $\zeta$  increases in the right-handed sense about the  $h$  axis

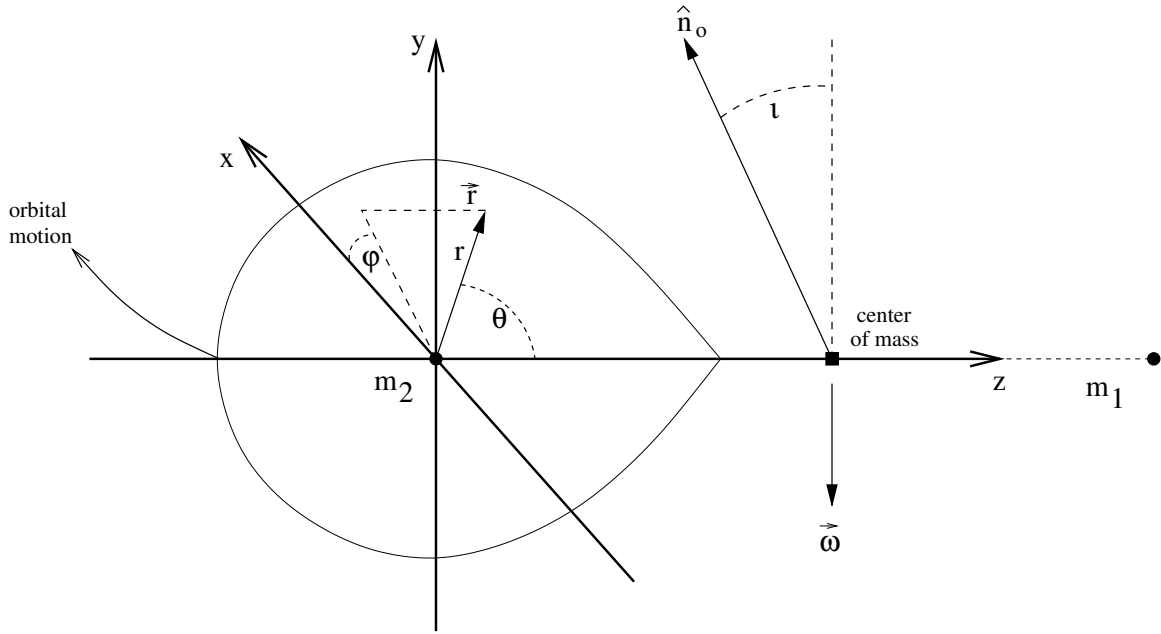


Figure 9: The spherical polar and Cartesian coordinate systems centered on the secondary star.

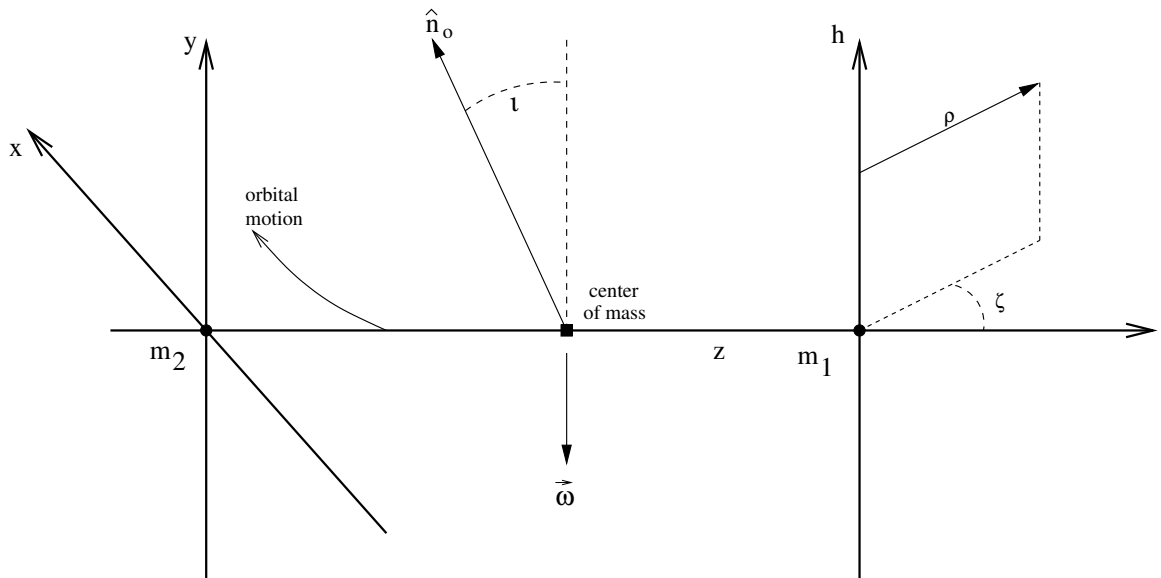


Figure 10: The cylindrical coordinate system centered on the primary star, and its relation to the Cartesian coordinate system centered on the secondary star.

In spherical polar coordinates the explicit forms of  $r_1$ ,  $r_2$ , and  $\rho_{\text{cm}}$  used in Section III.1 are

$$\begin{aligned} r_1^2 &= a_s^2 + r^2 - 2a_s r \cos \theta = a_s^2 + r^2 - 2a_s r \text{n} \\ r_2 &= r \\ \rho_{\text{cm}}^2 &= r^2 \sin^2 \theta \cos^2 \phi + (r \cos \theta - r_{\text{cm}})^2 = r^2 \text{l}^2 + (r \text{n} - r_{\text{cm}})^2 \end{aligned} \quad (\text{A.1})$$

where angles  $\theta$  and  $\phi$  and direction cosines  $\text{l}$  and  $\text{n}$  are defined by equations A.4, and  $r_{\text{cm}}$  is the distance from the center of star 2 to the center of mass.

The unit vector pointing toward the Earth is  $\hat{n}_o$ . The orbital inclination,  $i$ , is the angle between  $\hat{n}_o$  and the  $y$  axis of the Cartesian coordinate system (or, alternatively, the angle between  $\hat{n}_o$  and the negative rotation axis). In Cartesian coordinates  $\hat{n}_o$  is given by

$$\hat{n}_o = -\hat{i} \sin i \sin p + \hat{j} \cos i - \hat{k} \sin i \cos p \quad (\text{A.2})$$

where  $(\hat{i}, \hat{j}, \hat{k})$  are the unit vectors along the coordinate axes, and  $p$  is the orbital phase. Note that  $\hat{n}_o$  rotates counterclockwise(!) around the  $y$  axis, and  $p = 0$  when  $\hat{n}_o$  points in the  $-z$  direction. Phase zero is, therefore, the phase of inferior conjunction of star 2 (star 2 in front of star 1). Note that  $i = 90^\circ$  when the observer is in the plane of the orbit.

## A.2. Transformations between Cartesian and Spherical Polar Coordinates

Let  $(x, y, z)$  and  $(r, \theta, \phi)$  be the usual coordinates in the Cartesian and spherical polar coordinate systems. The transformations from  $(r, \theta, \phi)$  to  $(x, y, z)$  coordinates are

$$\begin{aligned} x &= r \text{l} = r \sin \theta \cos \phi \\ y &= r \text{m} = r \sin \theta \sin \phi \\ z &= r \text{n} = r \cos \theta \end{aligned} \quad (\text{A.3})$$

where  $\text{l}$ ,  $\text{m}$ , and  $\text{n}$  are the direction cosines:

$$\begin{aligned} \text{l} &= \sin \theta \cos \phi \\ \text{m} &= \sin \theta \sin \phi \\ \text{n} &= \cos \theta \end{aligned} \quad (\text{A.4})$$

The reverse transformations are

$$\begin{aligned} r^2 &= x^2 + y^2 + z^2 \\ \cos \theta &= \frac{z}{(x^2 + y^2 + z^2)^{\frac{1}{2}}} \\ \cos \phi &= \frac{x}{(x^2 + y^2)^{\frac{1}{2}}} \\ \sin \phi &= \frac{y}{(x^2 + y^2)^{\frac{1}{2}}} \end{aligned} \quad (\text{A.5})$$

In the Cartesian coordinate system a vector  $\vec{A}$  is written

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \quad (\text{A.6})$$

where  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$  are the unit vectors in the  $x$ ,  $y$ , and  $z$  directions, and  $A_x$ ,  $A_y$ , and  $A_z$  are the components of the vector; and in spherical polar coordinates the vector is written

$$\vec{A} = A_r \hat{r} + A_\theta \hat{\theta} + A_\phi \hat{\phi} \quad (\text{A.7})$$

where  $\hat{r}$ ,  $\hat{\theta}$ , and  $\hat{\phi}$  are unit vectors in the  $r$ ,  $\theta$ , and  $\phi$  directions, and  $A_r$ ,  $A_\theta$ , and  $A_\phi$  are the components of the vector. The unit vectors transform by

$$\begin{aligned} \hat{i} &= \hat{r} \sin \theta \cos \phi + \hat{\theta} \cos \theta \cos \phi - \hat{\phi} \sin \phi \\ \hat{j} &= \hat{r} \sin \theta \sin \phi + \hat{\theta} \cos \theta \sin \phi + \hat{\phi} \cos \phi \\ \hat{k} &= \hat{r} \cos \theta - \hat{\theta} \sin \theta \end{aligned} \quad (\text{A.8})$$

and, therefore, the components transform by

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 0 \end{pmatrix} \begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} \quad (\text{A.9})$$

$$\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} = \begin{pmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad (\text{A.10})$$

and the values of the trigonometric functions in equation A.10 come from equations A.5.

### A.3. Transformations between Cartesian and Cylindrical Coordinates

Let  $(\rho, \zeta, h)$  be the cylindrical coordinates. The orientation and displacement of the cylindrical coordinate system from the Cartesian coordinate system are shown in Figure 10. The transformations from  $(\rho, \zeta, h)$  to  $(x, y, z)$  coordinates are

$$\begin{aligned} x &= \rho \sin \zeta \\ y &= h \\ z &= \rho \cos \zeta + a \end{aligned} \quad (\text{A.11})$$

where  $a$  is the separation of the centers of mass of the two stars. The reverse transformations are

$$\begin{aligned} \rho^2 &= x^2 + (z - a)^2 \\ \cos \zeta &= \frac{z - a}{[x^2 + (z - a)^2]^{\frac{1}{2}}} \\ \sin \zeta &= \frac{x}{[x^2 + (z - a)^2]^{\frac{1}{2}}} \\ h &= y \end{aligned} \quad (\text{A.12})$$

A vector in cylindrical coordinates a vector is written

$$\hat{A} = A_\rho \hat{\rho} + A_\zeta \hat{\zeta} + A_h \hat{h} \quad (\text{A.13})$$

The transformations of the components of vectors between cylindrical and Cartesian coordinates are then

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \sin \zeta & \cos \zeta & 0 \\ 0 & 0 & 1 \\ \cos \zeta & -\sin \zeta & 0 \end{pmatrix} \begin{pmatrix} A_\rho \\ A_\zeta \\ A_h \end{pmatrix} \quad (\text{A.14})$$

$$\begin{pmatrix} A_\rho \\ A_\zeta \\ A_h \end{pmatrix} = \begin{pmatrix} \sin \zeta & 0 & \cos \zeta \\ \cos \zeta & 0 & -\sin \zeta \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad (\text{A.15})$$

#### A.4. The Gradient Operator

The explicit form of the gradient operator in the Cartesian coordinate system is

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}, \quad (\text{A.16})$$

where  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$  are the unit vectors in the  $x$ ,  $y$ , and  $z$  directions. Its explicit form in the cylindrical coordinate system is

$$\nabla = \hat{\rho} \frac{\partial}{\partial \rho} + \hat{\zeta} \left( \frac{1}{\rho} \right) \frac{\partial}{\partial \zeta} + \hat{h} \frac{\partial}{\partial h}, \quad (\text{A.17})$$

where  $\hat{\rho}$ ,  $\hat{\zeta}$ , and  $\hat{h}$  are the unit vectors in the  $\rho$ ,  $\zeta$ , and  $h$  directions. Its form in the spherical polar coordinate system is

$$\nabla = \hat{r} \frac{\partial}{\partial r} + \hat{\theta} \left( \frac{1}{r} \right) \frac{\partial}{\partial \theta} + \hat{\phi} \left( \frac{1}{r \sin \theta} \right) \frac{\partial}{\partial \phi}, \quad (\text{A.18})$$

where  $\hat{r}$ ,  $\hat{\theta}$ , and  $\hat{\phi}$  are the unit vectors in the  $r$ ,  $\theta$ , and  $\phi$  directions.

#### A.5. The Normal Vector

In Cartesian coordinates the normal vector is

$$\hat{n} = n_x \hat{x} + n_y \hat{y} + n_z \hat{z}. \quad (\text{A.19})$$

Referring to Figure 11, we have

$$\frac{n_x}{n_z} = -\frac{\partial f}{\partial x} \quad (\text{A.20})$$

or

$$n_x = -\frac{\partial f}{\partial x} n_z \quad (\text{A.21})$$

A similar relation holds for  $n_y$ :

$$n_y = -\frac{\partial f}{\partial y} n_z \quad (\text{A.22})$$

and thus we have

$$\hat{n} = -\frac{\partial f}{\partial x} n_z \hat{x} - \frac{\partial f}{\partial y} n_z \hat{y} + n_z \hat{z} \quad (\text{A.23})$$

In fact, we do not know  $n_z$ , so we actually have

$$\hat{n} = \beta \left[ -\frac{\partial f}{\partial x} \hat{x} - \frac{\partial f}{\partial y} \hat{y} + \hat{z} \right] \quad (\text{A.24})$$

where  $\beta$  is set by the normalization condition  $|\hat{n}| = 1$ , giving

$$\beta = \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 + 1 \right]^{-1/2} \quad (\text{A.25})$$

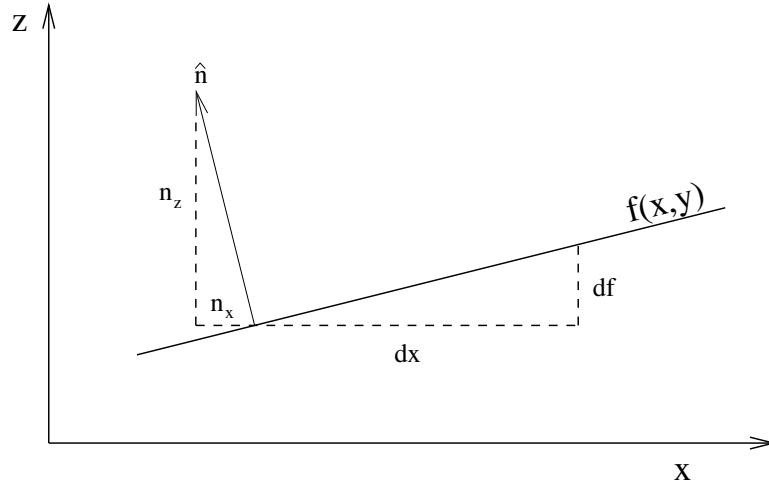


Figure 11: A cut through the surface defined by the function  $z = f(x, y)$ . The cut is parallel to the  $(x, z)$  plane. The vector  $\hat{n}$  is normal to the surface. The projection of the normal onto the plane of the cut has components  $n_x$  and  $n_z$ .

Equation A.24 can be generalized to an arbitrary coordinate system by defining an auxiliary function  $V$ :

$$V = z - f(x, y) \quad (\text{A.26})$$

and then the surface normal is given by

$$\hat{n} = \frac{\nabla V}{|\nabla V|} \quad (\text{A.27})$$

Note that all of the derivatives in equation A.27 must remain finite over the region of interest.

For the surface  $h = g(\rho, \zeta)$  in the cylindrical coordinate system  $(\rho, \zeta, h)$ , equation A.27 yields

$$\hat{n} \propto -\frac{\partial g}{\partial \rho} \hat{\rho} - \frac{1}{\rho} \frac{\partial g}{\partial \zeta} \hat{\zeta} + \hat{h} \quad (\text{A.28})$$

with an obvious normalization factor.

### A.6. The Surface Element

For the surface in the  $(x, y, z)$  coordinate system shown in Figure 11 the element of surface area is

$$d\vec{A} = \frac{\hat{n} dS}{\hat{n} \cdot \hat{z}} \quad (\text{A.29})$$

where  $dS$  is an element of area in a flat plane parallel to the  $(x, y)$  coordinate plane:

$$dS = dx dy \quad (\text{A.30})$$

The factor  $1/(\hat{n} \cdot \hat{z})$  is the projection factor from the surface to the flat plane. Carrying out the operations explicitly we find

$$d\vec{A} = \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 + 1 \right]^{1/2} \hat{n} dx dy \quad (\text{A.31})$$

or, equivalently,

$$d\vec{A} = \left( -\frac{\partial f}{\partial x} \hat{x} - \frac{\partial f}{\partial y} \hat{y} + \hat{z} \right) dx dy \quad (\text{A.32})$$

and either form can be used, depending on which is more convenient. In cylindrical coordinates the element of surface area is

$$d\vec{A} = \left[ \left( \frac{\partial g}{\partial \rho} \right)^2 + \left( \frac{1}{\rho} \frac{\partial g}{\partial \zeta} \right)^2 + 1 \right]^{1/2} \hat{n} \rho d\rho d\zeta \quad (\text{A.33})$$

or, equivalently,

$$d\vec{A} = \left( -\frac{\partial g}{\partial \rho} \hat{\rho} - \frac{1}{\rho} \frac{\partial g}{\partial \zeta} \hat{\zeta} + \hat{h} \right) \rho d\rho d\zeta \quad (\text{A.34})$$



## Appendix B. The Restricted Three-Body Problem and Zero-Velocity Surfaces

### B.1. The Basic Equation of the Restricted Three-body Problem

Assume that the only force acting between two particles, is Newtonian gravity:

$$\vec{F} = -\frac{Gm_1m_2\vec{r}}{|\vec{r}|^3} \quad (\text{B.1})$$

where  $\vec{F}$  is the force acting between the two particles,  $G$  is the gravitational constant,  $m_1$  is the mass of particle 1,  $m_2$  is the mass of particle 2, and  $\vec{r}$  is the vector distance between them. Under these conditions Kepler's third law holds:

$$G(m_1 + m_2) = \omega^2 a^3 \quad (\text{B.2})$$

$$\omega = \frac{2\pi}{P} \quad (\text{B.3})$$

where  $\omega$  is the mean orbital angular velocity,  $a$  is the separation of the particles, and  $P$  is the orbital period.

In the restricted three-body problem, particles 1 and 2 are constrained to move in circular orbits around each other and a third particle, with mass  $m$  much less than  $m_1$  and  $m_2$ , is moving under the influence of their gravity. The equation of motion for the third particle is

$$m\ddot{\vec{r}} = \vec{F}_1 + \vec{F}_2 = -\frac{Gmm_1\vec{r}_1}{|r_1|^3} - \frac{Gmm_2\vec{r}_2}{|r_2|^3} \quad (\text{B.4})$$

where  $\vec{r}_1$  and  $\vec{r}_2$  are the vectors from masses  $m_1$  and  $m_2$  to mass  $m$ .

It is convenient to use a coordinate system that has its origin at the center of mass and rotates with angular velocity  $\vec{\omega}$ , where  $\vec{\omega}$  is perpendicular to the orbital plane, so that  $m_1$  and  $m_2$  remain stationary in the coordinate system. The center of mass of the binary is at a distance

$$z_{\text{cm}} = \frac{m_1}{m_1 + m_2} a = \frac{1}{1 + q} a \quad (\text{B.5})$$

from mass  $m_2$ , where  $q = m_2/m_1$ .

To convert  $\ddot{\vec{r}}$  to the rotating coordinate system, use the relation

$$\frac{d\vec{A}}{dt} = \frac{d^*\vec{A}^*}{dt} + \vec{\omega} \times \vec{A}^* \quad (\text{B.6})$$

where the unstarred derivative is taken in the non-rotating coordinate system, the starred derivative is taken in the rotating coordinate system, and  $\vec{A}$  is an arbitrary vector, unstarred in the stationary frame and starred in the rotating frame. Successive applications of this relation yield

$$\ddot{\vec{r}} = \frac{d^2\vec{r}}{dt^2} = \frac{d^{*2}\vec{r}^*}{dt^2} + 2\vec{\omega} \times \frac{d^*\vec{r}^*}{dt} + \frac{d^*\vec{\omega}}{dt} \times \vec{r}^* + \vec{\omega} \times (\vec{\omega} \times \vec{r}^*) \quad (\text{B.7})$$

Since this is the restricted three-body problem,  $d\vec{\omega}/dt = d^*\vec{\omega}/dt = 0$ , and the equation of motion in the rotating frame simplifies to

$$m \frac{d^{*2}\vec{r}^*}{dt^2} + 2m\vec{\omega} \times \frac{d^*\vec{r}^*}{dt} + m\vec{\omega} \times (\vec{\omega} \times \vec{r}^*) = \vec{F}_1 + \vec{F}_2 \quad (\text{B.8})$$

From now on we will work in the rotating frame, so we can drop the asterisks without confusion and retrieve the standard form of the equation of motion for the restricted three-body problem

$$m\vec{r} = -\frac{Gmm_1\vec{r}_1}{|r_1|^3} - \frac{Gmm_2\vec{r}_2}{|r_2|^3} - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2m\vec{\omega} \times \vec{r} \quad (\text{B.9})$$

## B.2. Jacobi's Integral

Jacobi's Integral is essentially an energy integral. To derive it, use the cylindrical coordinate system with coordinates  $(\rho, \theta, z)$  specified by

- The origin is at the center of mass.
- The  $z$  axis is parallel to  $\vec{\omega}$
- $\rho$  is the perpendicular distance from the  $z$  axis.  $\vec{\rho} = \rho\hat{\rho}$ , where  $\hat{\rho}$  is the unit vector in the  $\rho$  direction.
- $\theta$  is the angle measured around  $z$  axis

In this coordinate system  $\vec{r}$  and  $\vec{\omega}$  are given by

$$\vec{r} = z\hat{z} + \vec{\rho} \quad (\text{B.10})$$

$$\vec{\omega} = \omega\hat{z} \quad (\text{B.11})$$

where  $\hat{z}$  is the unit vector in the  $z$  direction. The centrifugal force now simplifies to

$$\begin{aligned} -m\vec{\omega} \times (\vec{\omega} \times \vec{r}) &= -m\omega^2\hat{z} \times [\hat{z} \times (\hat{z} + \vec{\rho})] \\ &= -m\omega^2\hat{z} \times (\hat{z} \times \vec{\rho}) \\ &= +m\omega^2\vec{\rho} \end{aligned} \quad (\text{B.12})$$

and the equation of motion becomes

$$m\vec{r} = -\frac{Gmm_1\vec{r}_1}{|r_1|^3} - \frac{Gmm_2\vec{r}_2}{|r_2|^3} + m\omega^2\vec{\rho} - 2m\vec{\omega} \times \vec{r} \quad (\text{B.13})$$

Dot the velocity vector  $\vec{r}$  into both sides of the equation of motion. Since

$$(2m\vec{\omega} \times \vec{r}) \cdot \vec{r} = 0 \quad (\text{B.14})$$

$$\vec{\rho} \cdot \vec{r} = \vec{\rho} \cdot (z\hat{z} + \vec{\rho}) = \vec{\rho} \cdot \vec{\rho} \quad (\text{B.15})$$

the basic equation collapses to

$$m\vec{r} \cdot \ddot{\vec{r}} = -\frac{Gmm_1\vec{r}_1}{|r_1|^3} \cdot \vec{r} - \frac{Gmm_2\vec{r}_2}{|r_2|^3} \cdot \vec{r} + m\omega^2\vec{\rho} \cdot \vec{\rho} \quad (\text{B.16})$$

Using the identity  $\vec{r} = \vec{r}_1 = \vec{r}_2$ , this equation integrates to

$$\frac{d}{dt} \left( \frac{1}{2}m\vec{r} \cdot \dot{\vec{r}} \right) = \frac{d}{dt} \left( \frac{Gmm_1}{r_1} \right) + \frac{d}{dt} \left( \frac{Gmm_2}{r_2} \right) + \frac{d}{dt} \left( \frac{1}{2}m\omega^2\vec{\rho} \cdot \vec{\rho} \right) \quad (\text{B.17})$$

and, finally, we find Jacobi's integral

$$V = \frac{1}{2}v^2 - \frac{Gm_1}{r_1} - \frac{Gm_2}{r_2} - \frac{1}{2}\omega^2\rho^2 \quad (\text{B.18})$$

We recognize  $V$  as total energy per unit mass.

### B.3. Zero-Velocity Surfaces

For  $v = 0$ , Jacobi's integral reduces to

$$V = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2} - \frac{1}{2}\omega^2\rho^2 \quad (\text{B.19})$$

This is usually put in the form

$$V = \frac{Gm_1}{a} \left[ -\frac{a}{r_1} - \frac{qa}{r_2} - \frac{1}{2}(1+q)\left(\frac{\rho}{a}\right)^2 \right] \quad (\text{B.20})$$

where, again,  $q = m_2/m_1$  is the mass ratio. The loci of constant  $V$  are the zero-velocity surfaces.

## Appendix C. Calculation of $I_{\perp}$ and $\bar{I}_{\perp X}$

### C.1. Mean Specific Intensities through Filters

**Intensities for Theoretical Stellar Spectra:** The specific intensity perpendicular to the surface of a star,  $I_{\perp}$ , is not generally available in the literature, so it has been calculated from fluxes and limb darkening, which are available. On the other hand, the published descriptions of the way in which the mean fluxes through filter bandpasses have been calculated are inscrutable. To avoid misinterpretation, the mean fluxes have been calculated from scratch.

We first calculate the mean flux density in bandpass  $X$  from an element of the surface of a star:

$$\bar{F}_X = \int R_X(\lambda)F(\lambda)d\lambda, \quad (\text{C.1})$$

where  $F(\lambda)$  is the flux per unit area as a function of wavelength in ergs/cm/s/Å and  $R_X(\lambda)$  is the normalized response function for the bandpass:

$$\int R_X(\lambda)d\lambda = 1. \quad (\text{C.2})$$

For the fluxes, we adopt the Kurucz spectra in the STScI calibration database (*circa* 1992). These spectra are whole-disk fluxes as a function of wavelength typically sampled at 10 Å intervals over the wavelengths of interest. To be explicit: The fluxes tabulated in the Kurucz models have the same meaning as black body fluxes given by

$$B(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1} \quad (\text{C.3})$$

and in regions not heavily line-blanketed have values roughly similar to black body fluxes. Figure 12 compares the Kurucz spectrum for  $T = 6000$  K and  $\log g = 4.0$  to the flux from a black body at the same temperature calculated from equation C.3. The Kurucz spectra for a selection of other temperatures are shown in the top panel of Figure 13.

The filter transmissions for the Johnson UBV and Cousins RI filters have been taken from Bessell (1990, PASP, 102, 1181); and the transmissions for the JKL filters have been taken from Bessell & Brett (1988, PASP, 100, 1134). Note that the U and B passbands are actually the UX and BX passbands. Also note that the bottom few lines of Table IV in Bessell & Brett (1988) have a typo that displaces the entries by one column. The effective wavelengths of the bandpasses are given in the following table (Bessell 1988, 1990), where the effective wavelengths of the UBVRI filters are calculated for a K0III star and those of the JKL filters for an A0V star.

Filter:	U	B	V	R	I	J	H	K
$\lambda_{eff}$ :	3656 Å	4537 Å	5524 Å	6535 Å	8028 Å	1.22 μm	1.63 μm	2.19 μm

The passbands of the UBVRI filters are shown in the lower panel of Figure 13.

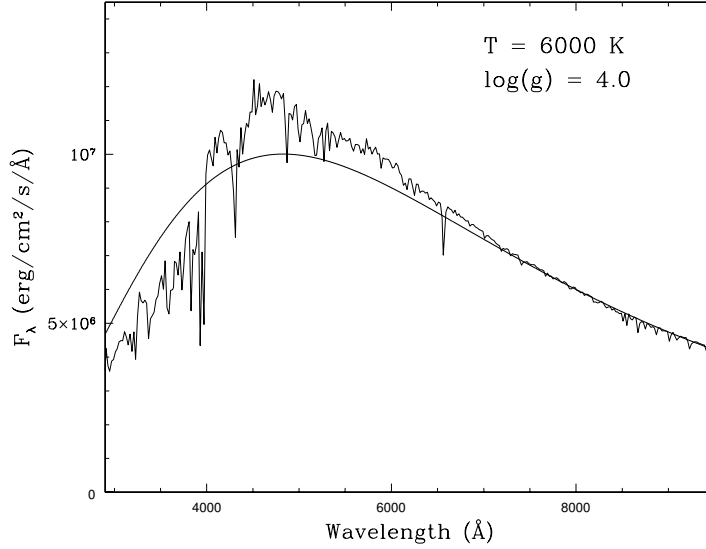


Figure 12: A comparison of the Kurucz spectrum for  $T = 6000$  K and  $\log g = 4.0$  to the flux from a black body at the same temperature calculated from equation C.3

The relation between flux and specific intensity at the surface of a star is

$$F(\lambda) = 2\pi \int_0^1 I(\lambda, \mu) \mu d\mu = 2\pi I_{\perp}(\lambda) \int_0^1 H(\mu) \mu d\mu, \quad (\text{C.4})$$

where  $\mu = \cos \theta$  is the angle between the light ray and the vector normal to the surface of the star. The second equality comes from setting:

$$I(\lambda, \mu) = I_{\perp}(\lambda) H(\mu), \quad (\text{C.5})$$

where  $I_{\perp}(\lambda)$  is the specific intensity perpendicular to the surface of the star,  $I_{\perp}(\lambda) = I(\lambda, 1)$ , and  $H(\mu)$  is the limb-darkening law, normalized so that  $H(1) = 1$ . Equation C.5 is in essence the definition of limb darkening.

If one wants the monochromatic fluxes and intensities, equation C.4 is the final result since, after rearranging,

$$I_{\perp}(\lambda) = F(\lambda) / \left[ 2\pi \int_0^1 H(\mu) \mu d\mu \right]. \quad (\text{C.6})$$

We are, however, interested in the fluxes measured in a bandpass, and so we multiply equation C.4 by  $R_X(\lambda)$  and integrate over  $\lambda$  to get

$$\bar{F}_X = 2\pi \bar{I}_{\perp X} \int_0^1 H_X(\mu) \mu d\mu, \quad (\text{C.7})$$

where

$$\bar{I}_{\perp X} = \int R_X(\lambda) I_{\perp}(\lambda) d\lambda \quad (\text{C.8})$$

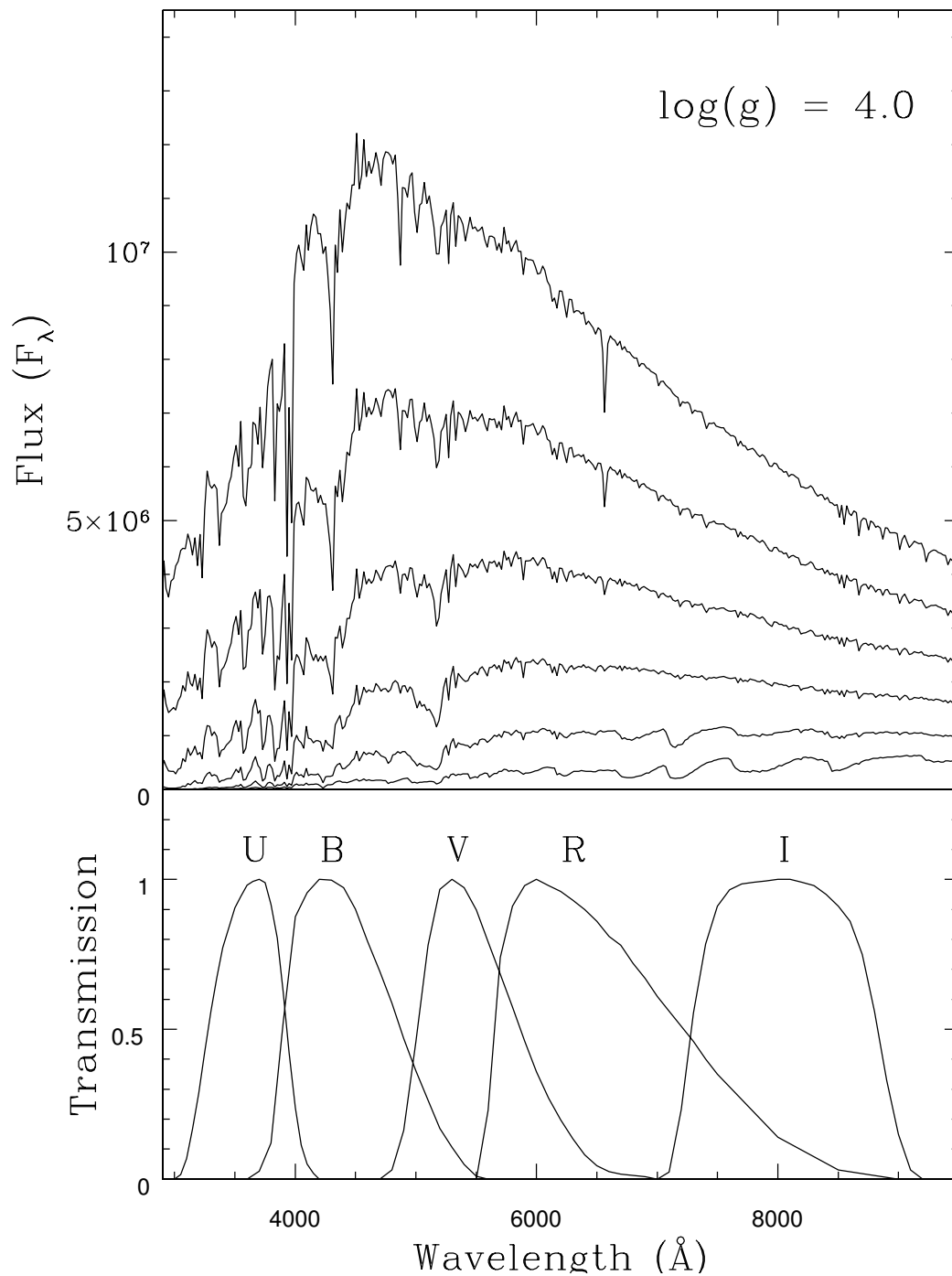


Figure 13: Lower panel: The response functions for the UBVRI passbands. Upper panel: A selection of Kurucz spectra used to calculate the mean flux densities. All the spectra have  $\log(g) = 4.0$ ; the temperatures are 3500 K, 4000 K, 4500 K, 5000 K, 5500 K, and 6000 K.

and so  $\bar{I}_{\perp X}$  is given by

$$\bar{I}_{\perp X} = \bar{F}_X / \left[ 2\pi \int_0^1 H_X(\mu) \mu d\mu \right]. \quad (\text{C.9})$$

Note that we have explicitly assumed that  $H(\mu)$  is independent of wavelength and equal to  $H_X(\mu)$  within bandpass X.

We adopt the limb darkening law proposed by Claret (2000, *Astron. & Astrophys.*, 363, 1081):

$$H_X(\mu) = \frac{\bar{I}_X(\mu)}{\bar{I}_{\perp X}} = 1 - a_1(1 - \mu^{1/2}) - a_2(1 - \mu) - a_3(1 - \mu^{3/2}) - a_4(1 - \mu^2). \quad (\text{C.10})$$

The coefficients  $a_i$  have been taken from the extensive tables for various filters published by Claret in electronic form on the CDS. Note that the integral of the Claret law can be evaluated analytically:

$$\int_0^1 H_X(\mu) \mu d\mu = 2 \left[ \frac{1}{4}(1 - a_1 - a_2 - a_3 - a_4) + \frac{a_1}{5} + \frac{a_2}{6} + \frac{a_3}{7} + \frac{a_4}{8} \right]. \quad (\text{C.11})$$

**Intensities for Black Body Spectra:** For a black body spectrum, the calculation is somewhat simpler because the specific intensity is independent of  $\mu$  (there is no limb darkening):

$$\frac{dI_X(\mu)}{d\mu} = 0 \quad (\text{C.12})$$

and then, from equation C.10

$$H_X(\mu) = \frac{\bar{I}_X(\mu)}{\bar{I}_{\perp X}} = 1. \quad (\text{C.13})$$

Equation C.6 becomes

$$I_{\perp}(\lambda) = \frac{1}{\pi} B(\lambda) \quad (\text{C.14})$$

and we have the obvious result

$$\bar{I}_X(\mu) = \bar{I}_{\perp X} = \frac{1}{\pi} \int R_X(\lambda) B(\lambda) d\lambda \quad (\text{C.15})$$

Note that the mean intensity through filter X calculated from equation C.15 will not agree with the intensity calculated from equation C.9 even where a black body is a good approximation to the fluxes in a stellar spectrum because the black body intensity is not limb darkened.

## C.2. Mean Specific Intensities in a Rectangular (Sharp-Edged) Bandpass

For a black body spectrum, the mean specific intensity in a bandpass from  $\lambda_a$  to  $\lambda_b$  is

$$\bar{I}_{ab} = \frac{1}{\pi(\lambda_b - \lambda_a)} \int_{\lambda_a}^{\lambda_b} B(\lambda) d\lambda \quad (\text{C.16})$$

Defining

$$\zeta = \frac{hc}{\lambda kT}, \quad (\text{C.17})$$

we recast equation C.16 into the form

$$\bar{I}_{ab} = \frac{T^4}{\lambda_b - \lambda_a} [Z(\zeta_b) - Z(\zeta_a)] \quad (\text{C.18})$$

where

$$Z(\zeta) = \frac{2k^4}{h^3c^2} \int_0^\zeta \frac{x^3 dx}{\exp(x) - 1}. \quad (\text{C.19})$$

For convenience and speed, the function  $Z(\zeta)$  is calculated off line and saved in a table. Note that  $Z(\infty) = \sigma/\pi = 1.8049 \times 10^{-5}$ , which may be useful as an error check.



## Appendix D. Description of the Input Files and Parameters

XRbinary was written for a Linux operating system but should run under most Unix-like operating systems. XRbinaryV2.x is invoked by the command

XRbinaryV2.x.exe **parfilename**

where **parfilename** is the name of the file specifying the model parameters. The program writes two output files whose names are based on **parfilename**:

**parfilename.SysPars**: The model parameters input from **parfilename** plus some other useful information about the model. It also records the value of  $\chi^2$  if the model light curve was fitted to an observed light curve.

**parfilename.LC**: The light curves calculated from the model.

### D.1. The Input Files

The XRbinary program requires five tables providing information about specific intensities and gravity darkening. These tables will generally not need to be modified by the user.

**GTable.dat**: The gravity-darkening coefficients as a function of temperature.

**LDTable.dat**: The limb darkening coefficients for realistic spectra observed through filters. This table is the companion to the table in **IperpTable.dat**.

**IperpTable.dat**: The specific intensity perpendicular to the surface element for realistic spectra observed through filters.

**IBBfilterTable.dat**: The mean specific intensities of black bodies observed through filters.

**ZzetaTable.dat**: A table for use in calculating the mean intensities of black bodies observed in a square-edged bandpass.

The parameter file **parfilename** consists of a series of keywords, one keyword per line (see section D.2 below for an example of a parameter file). Most keywords are followed by a list of numbers or words. The keywords and the meanings of their parameters are:

Miscellaneous keywords:

**COMMENT=** Comment line

blank line blank lines have no effect

**END** Terminates reading of the parameter file.

**VERBOSE=** ON or OFF

If ON, the program prints out information about what it is doing.

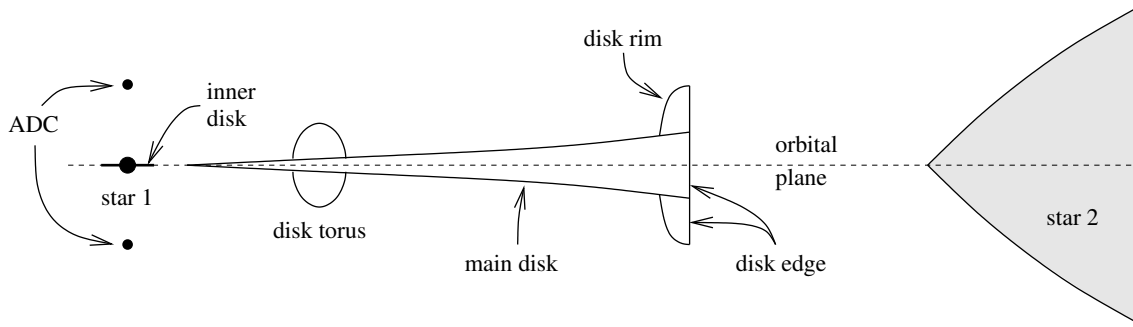


Figure D.1: The various components of the XRbinary model.

DIAGNOSTICS= OFF

or

*keyword orbital-phase*

If OFF, do not print diagnostic information.

If *keyword*, execute a diagnostic procedure. The diagnostic procedures generally write information to files with a suffix “.inspect” and then halt the program.

Some available keywords:

NOCHECKPARS  
 INSPECTINPUT  
 INSPECTSYSPARS  
 INSPECTSTAR2TILES  
 INSPECTDISKTILES  
 INSPECTYLIMITS  
 INSPECTHEATING  
 INSPECTESCAPE

The INSPECTESCAPE keyword also requires an orbital phase, which is specified by *orbital-phase*.

Keywords that control the basic properties of the model (Figure D.1 shows the components of the XRbinary model):

STAR1= ON or OFF

If ON, the primary star emits flux.

STAR2= ON or OFF

If ON, the secondary star emits flux.

STAR2SPOTS= ON or OFF

If ON, the secondary star can have up to 19 spots.

DISK= ON or OFF

If ON, the model contains an accretion disk.

DISKRIM= ON or OFF

If ON, the disk has a raised rim.

DISKTORUS= ON or OFF  
 If ON, the disk has a raised torus at an intermediate radius.

INNERDISK= ON or OFF  
 If ON, the disk has a small, flat, inner disk.

DISKSPOTS= ON or OFF  
 If ON, the disk can have up to 19 spots.

ADC= ON or OFF  
 If ON, the disk has an ADC.

THIRDLIGHT= ON or OFF  
 If ON, the light curve has flux from a “third light.”

IRRADIATION= ON or OFF  
 If ON, the model includes heating by irradiation.

Keywords referring to the orbital light curve:

PHASES= *phasemin phasemax deltaphase*  
 The phase coverage and resolution of the orbital light curves in fractions of the orbit. Phase 0.0 is the phase of inferior conjunction of the secondary star. The phases must be greater than -0.5 and less than 1.0.

PHASEOFFSET= *phaseoffset*  
 The offset of the orbital light curves in fractions of the orbit. The light curves are calculated for (phase - *deltaphase*), ie, the light curves are moved forward in phase by *deltaphase*.

BANDPASS= FILTER *filtername*  
 or  
 SQUARE *minwavelength maxwavelength*  
 If FILTER, calculate the lightcurve for the filter with name *filtername*. Typical filter names are U, B, V, R, I, J, H, K etc.  
 If SQUARE, calculate the lightcurve in a bandpass bounded by *minwavelength* and *maxwavelength*. The wavelengths are in Ångstroms.  
 The program can calculate light curves in many bandpasses at once, so *parfile.dat* can contain many lines beginning with this keyword.

NORMALIZE= OFF  
 or  
 MAXVALUE *maxvalue*  
 If OFF, do not normalize the light curves  
 If MAXVALUE, normalize the flux to *maxvalue* at the phase of maximum flux.  
 or  
 FITDATA *filtername*  
 or

### FITDATA SQUARE *minlambda maxlambda*

Normalize the light curve by calculating the multiplicative factor that minimizes the variance between the calculated and observed light curves in either filter *filtername* or the square band-pass bounded by wavelengths *minlambda* and *maxlambda*. The wavelengths are in Ångstroms. The same normalization factor is then applied to all the calculated light curves. The pertinent observed light curve must be read in by a READDATA= statement.

Keywords referring to the whole system:

PERIOD=	<i>period</i>	The orbital period in days.
K2=	<i>K2</i>	The amplitude of the projected radial velocity curve in km/s. Either K2 or M1 can be specified, but not both.
M1=	<i>M1</i>	The mass of star 1 in solar masses. Either K2 or M1 can be specified, but not both.
MASSRATIO=	<i>massratio</i>	The mass ratio, $q = m_2/m_1$ .
INCLINATION=	<i>inclination</i>	The orbital inclination in degrees.

Keywords referring to star 1:

STAR1LUM=	<i>luminosity</i>	The luminosity of star 1 in ergs/sec.
STAR1TEMP=	<i>temperature</i>	The effective temperature of star 1 in Kelvins.

Keywords referring to star 2:

STAR2TILES=	<i>targetNtiles</i>	The target number of tiles with which to cover the surface of star 2. The maximum number of tiles is 50,000; 5000 tiles should give good results.
STAR2TEMP=	<i>meanTemperature</i>	The mean temperature of star 2 in Kelvins.
STAR2ALBEDO=	<i>albedo</i>	The albedo of star 2.

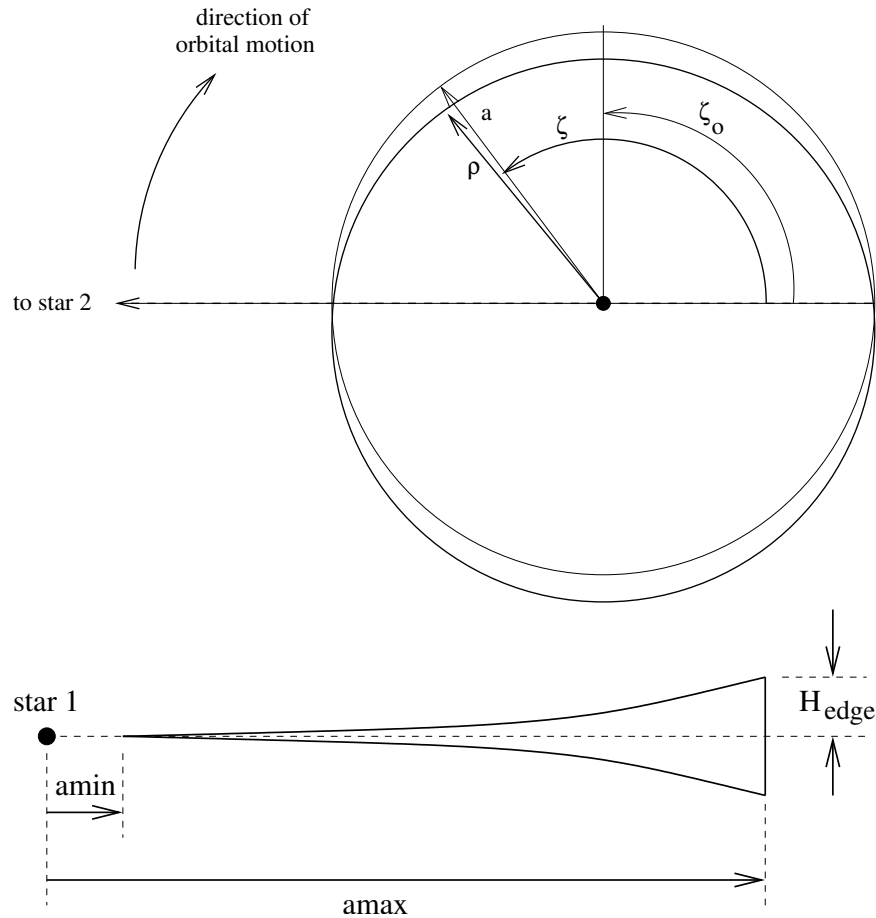


Figure D.2: Top and side views of the eccentric disk. All disk properties are specified as if the disk were circular with radial coordinate  $a$ . The disk is then “stretched” to make it elliptical by means of the transformation  $\rho = a(1 - e^2)/[1 + e \cos(\zeta - \zeta_0)]$ , where  $\zeta_0$  is the longitude of periastron.

STAR2SPOT=

*theta phi radius Tratio*

*theta* and *phi* are the coordinates of the spot center in the spherical polar coordinate system. *radius* is the angular radius of the spot as seen from the center of the secondary star. The unit is degrees for all three.

*Tratio* is the ratio of the spot temperature to the local effective temperature. If less than 1, the spot is cooler than the unspotted star; and if greater than 1, it is hotter.

This keyword can appear multiple times, once for each spot.

Keywords referring to the accretion disk (see Figure D.2 shows the meaning of the parameters for the eccentric disk):

Note: All lengths are input as a fraction of the separation of the centers of mass of the two stars.

- DISKTILES= *targetNtiles*  
The target number of tiles with which to cover the surface of the disk. The maximum number of tiles is 50,000; 20,000 tiles should give good results.
- DISKE= *eccentricity*  
The eccentricity of the elliptical disk.
- DISKZETAZERO= *periastron-angle*  
The angle of the periastron of the elliptical disk in degrees.
- DISKALBEDO= *albedo*  
The albedo of star 2.

Figure D.2 shows the meaning of the parameters for the main disk.

- MAINDISKA= *amin amax*  
The minimum and maximum semi-major axis of the main disk.
- MAINDISKH= *Hedge Hpow*  
The parameters in the function for the main disk height:

$$H_{main} = H_{edge} \left( \frac{a - a_{min}}{a_{max} - a_{min}} \right)^{H_{pow}} .$$

- MAINDISKT= VISCOUS *maindiskL*  
The temperature distribution on the main disk will be that of a steady-state, optically-thick, viscous disk (see equation 28).  
*maindiskL* is the total luminosity of the disk.

or

- POWERLAW *Tpow maindiskL*  
The temperature distribution on the main disk will be a power law in disk radius:

$$T_{main} = K a^{T_{pow}} ,$$

with  $K$  set so that the total luminosity of the main disk is *maindiskL*.

- DISKEDGET= *Tedge Tspot ZetaMid ZetaWidth*  
*Tedge* is the temperature of the edge of the disk. The spot has temperature *Tspot*. If  $T_{spot} > T_{edge}$ , there is a spot on the edge of the disk and the spot extends over

$$(ZetaMid - ZetaWidth/2) \Rightarrow (ZetaMid + ZetaWidth/2).$$

Both angles are in degrees.

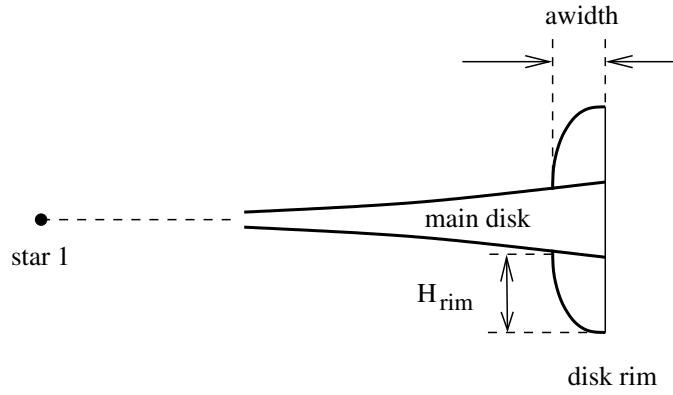


Figure D.3: Side view of the disk and disk rim, defining the meaning of the rim parameters.

DISKSPOT= *ZetaMin ZetaMax aMin aMax Tratio*  
*ZetaMin* and *ZetaMax* are the minimum and maximum angles over which the spot extends. If *ZetaMin* is greater than *ZetaMax*, the spot extends from *ZetaMin* through  $\zeta = 0$  to *ZetaMax*.

*aMin* and *aMax* are the minimum and maximum values of *a* over which the spot extends. If the spot extends past the inner or outer edge of the disk, it is truncated at the edge.

*Tratio* is fractional change in the disk temperature in the spot. If less than 1, the spot is cooler than the unspotted disk; and if greater than 1, it is hotter.

INNERDISKL= *luminosity*  
The luminosity of the inner disk in ergs/sec.

INNERDISKT= *temperature*  
The effective temperature of the inner disk in Kelvins.

Figure D.3 shows the meaning of the parameters for the disk rim.

DISKRIMAWIDTH= *awidth*  
The rim height has the functional form

$$H_{rim} = H_{rim}(\zeta) \left[ 1 - \left( \frac{a_{max} - a}{a_{width}} \right)^2 \right]^{1/2}$$

where the width of the rim is *awidth* and the outer edge is the same as the outer edge of the disk (*a<sub>max</sub>*).

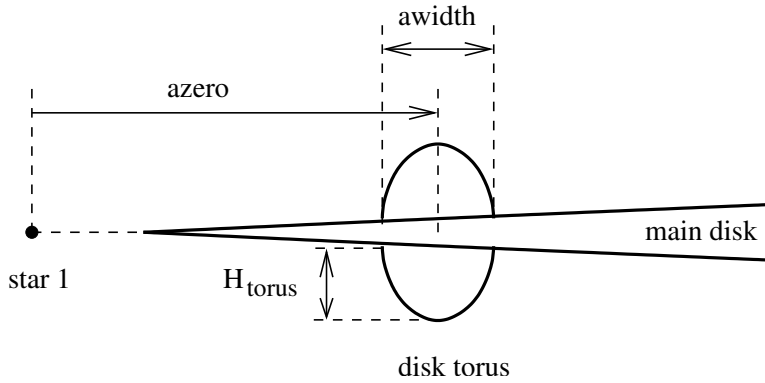


Figure D.4: Side view of the disk and disk torus, defining the meaning of the torus parameters.

DISKRIMPARS= SINUSOID  $H_{max}$   $H_{min}$   $Zeta_{H_{max}}$   $T_{max}$   $T_{min}$   $Zeta_{T_{max}}$

or

POINT  $Zeta$   $H_{zeta}$   $T_{zeta}$

If SINUSOID, the disk rim height is a sinusoid with functional form

$$H(\zeta) = \frac{1}{2}(H_{max} + H_{min}) + \frac{1}{2}(H_{max} - H_{min}) \cos(\zeta - \zeta_{H_{max}})$$

and its temperature is also as sinusoid with functional form

$$T = \frac{1}{2}(T_{max} + T_{min}) + \frac{1}{2}(T_{max} - T_{min}) \cos(\zeta - \zeta_{T_{max}})$$

If POINT, the disk rim height and temperature are defined by a set of points, one point per DISKRIMPARS= line in the parameter file:

```
DISKRIMPARS= POINT Zeta1 H1 T1
DISKRIMPARS= POINT Zeta2 H2 T2
                ⋮           ⋮           ⋮
```

The Zetas do not need to be in any particular order. If the temperature at any point is less than the temperature of the underlying (spotted) main disk at that point, the temperature is set equal to the spotted main disk temperature.

Figure D.4 shows the meaning of the parameters for the disk torus.

```
DISKTORUSAZERO= azero
DISKTORUSAWIDTH= awidth
```



The torus height has the functional form

$$H_{torus} = H_{torus}(\zeta) \left[ 1 - \left( \frac{a_{zero} - a}{(a_{width}/2)} \right)^2 \right]^{1/2}$$

where the torus is centered on  $a_{zero}$  and has width  $a_{width}$  (note that the meaning of  $a_{width}$  is different for the torus and the rim.

DISKTORUSPARS= SINUSOID  $H_{max}$   $H_{min}$   $Zeta$   $H_{max}$   $T_{max}$   $T_{min}$   $Zeta$   $T_{max}$   
or  
POINT  $Zeta$   $H_{zeta}$   $T_{zeta}$   
If SINUSOID, the disk torus height is a sinusoid with functional form

$$H(\zeta) = \frac{1}{2}(H_{max} + H_{min}) + \frac{1}{2}(H_{max} - H_{min}) \cos(\zeta - \zeta_{H_{max}})$$

and its temperature is also as sinusoid with functional form

$$T = \frac{1}{2}(T_{max} + T_{min}) + \frac{1}{2}(T_{max} - T_{min}) \cos(\zeta - \zeta_{T_{max}})$$

If POINT, the disk torus height and temperature are defined by a set of points, one point per DISKTORUSPARS= line in the parameter file:

```
DISKTORUSPARS= POINT Zeta1 H1 T1
DISKTORUSPARS= POINT Zeta2 H2 T2
                ⋮           ⋮   ⋮   ⋮
                ⋮           ⋮   ⋮   ⋮
```

The Zetas do not need to be in any particular order. If the temperature at any point is less than the temperature of the underlying (spotted) main disk at that point, the temperature is set equal to the spotted main disk temperature.

Keywords referring to the accretion disk corona (ADC):

ADCLUM= *luminosity*  
The luminosity of the ADC in ergs s<sup>-1</sup>, of which half is emitted above and half below the main disk.

ADCHEIGHT= *height*  
The distance of the ADC points above and below the orbital plane in units of the orbital separation.

Keywords referring to the flux from a “third light:”

3rdLIGHTPHASE= *thirdlight-orbphase*  
The orbital phase at which the amount of flux from the third light is specified.

3rdLIGHTFRACTION= *thirdlight-fraction*  
The fraction of the total flux at the specified orbital phase that is due to the third light.

Keywords referring to reading files with observed light curves:

READDATA= FILTER *filtername filename*  
or  
SQUARE *minlambda maxlambda filename*  
Read a file with an observed light curve in either filter *filtername* or the square bandpass bounded by wavelengths *minlambda* and *maxlambda*. The wavelengths are in Ångstroms. The format of the light curve files must be

phase1	flux1	stand-dev1
phase2	flux2	stand-dev2
⋮	⋮	⋮

and the standard deviations must all be present or all absent.

## D.2. A Typical parfile.dat

COMMENT= A typical parameter file for program XRbinaryV2.4.

```
VERBOSE=      ON
DIAGNOSTICS=  OFF

STAR1=        ON
STAR2=        ON
STAR2SPOTS=   ON
DISK=         ON
DISKRIM=      ON
DISKTORUS=   ON
INNERDISK=   ON
DISKSPOTS=    OFF
ADC=          ON
THIRDLIGHT=  OFF
IRRADIATION=  ON

PHASES=       -0.5  0.5  0.01
PHASEOFFSET=  0.007
BANDPASS=     FILTER  R
BANDPASS=     SQUARE 6300.0 6700.0
NORMALIZE=    FITDATA R
COMMENT= NORMALIZE= FITDATA SQUARE 6300.0 6700.0
COMMENT= NORMALIZE= MAXVALUE 1.0

PERIOD=       0.2222
COMMENT= K2=           300.0
M1=           1.34
MASSRATIO=    0.40
INCLINATION=  80.0

STAR1LUM=     1.5e36
STAR1TEMP=    3.0e5

STAR2TILES=   10000
STAR2TEMP=    4500.0
STAR2ALBEDO=  0.5

STAR2SPOT=    45.0 180.0 15.0 0.4
STAR2SPOT=    135.0 180.0 10.0 0.8

DISKTILES=    30000
DISKE=        0.1
```

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DISKZETAZERO= 30.0
DISKALBEDO= 1.0;

MAINDISKA= 0.001 0.30
MAINDISKH= 0.02 1.2
MAINDISKT= VISCOUS 1.0e37
COMMENT= MAINDISKT= POWERLAW -0.5 1.0e37

DISKEDGET= 5000.0 15000.0 110.0 20.0

DISKSPOT= 10.0 30.0 0.20 0.30 0.5
DISKSPOT= 120.0 150.0 0.10 0.20 1.5

INNERDISKL= 1.5e36
INNERDISKT= 3.0e5

DISKRIMAWIDTH= 0.04
DISKRIMPARS= POINT 0.0 0.06 15000.0
DISKRIMPARS= POINT 90.0 0.0 0.0
DISKRIMPARS= POINT 180.0 0.0 0.0
DISKRIMPARS= POINT 190.0 0.07 15000.0
DISKRIMPARS= POINT 270.0 0.0 0.0
COMMENT= DISKRIMPARS= SINUSOID 0.07 0.0 220.0 15000.0 5000.0 220.0

DISKTORUSAZERO= 0.10
DISKTORUSAWIDTH= 0.04
DISKTORUSPARS= POINT 0.0 0.0 0.0
DISKTORUSPARS= POINT 90.0 0.0 0.0
DISKTORUSPARS= POINT 170.0 0.0 0.0
DISKTORUSPARS= POINT 180.0 0.3 1500.0
DISKTORUSPARS= POINT 190.0 0.0 0.0
DISKTORUSPARS= POINT 270.0 0.0 0.0
COMMENT= DISKTORUSPARS= SINUSOID 0.07 0.0 220.0 15000.0 5000.0 220.0

ADCLUM= 2.0e36
ADCHEIGHT= 0.03

3rdLIGHTPHASE= 0.25
3rdLIGHTFRACTION= FILTER R 0.4
3rdLIGHTFRACTION= SQUARE 6300.0 6700.0 0.3

READDATA= SQUARE 6300.0 6700.0 Sqlightcurve.dat
READDATA= FILTER R Rlightcurve.dat

END

```