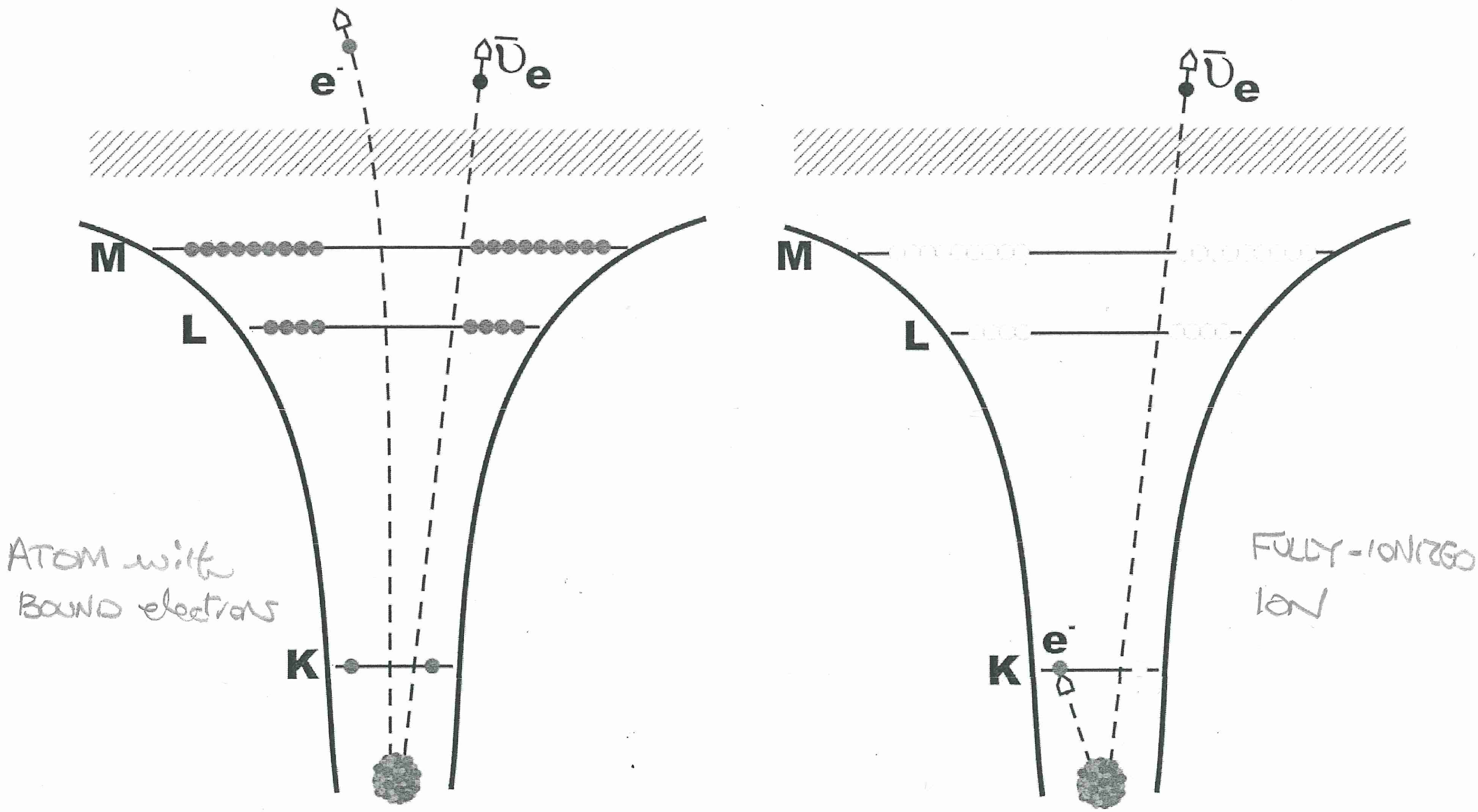
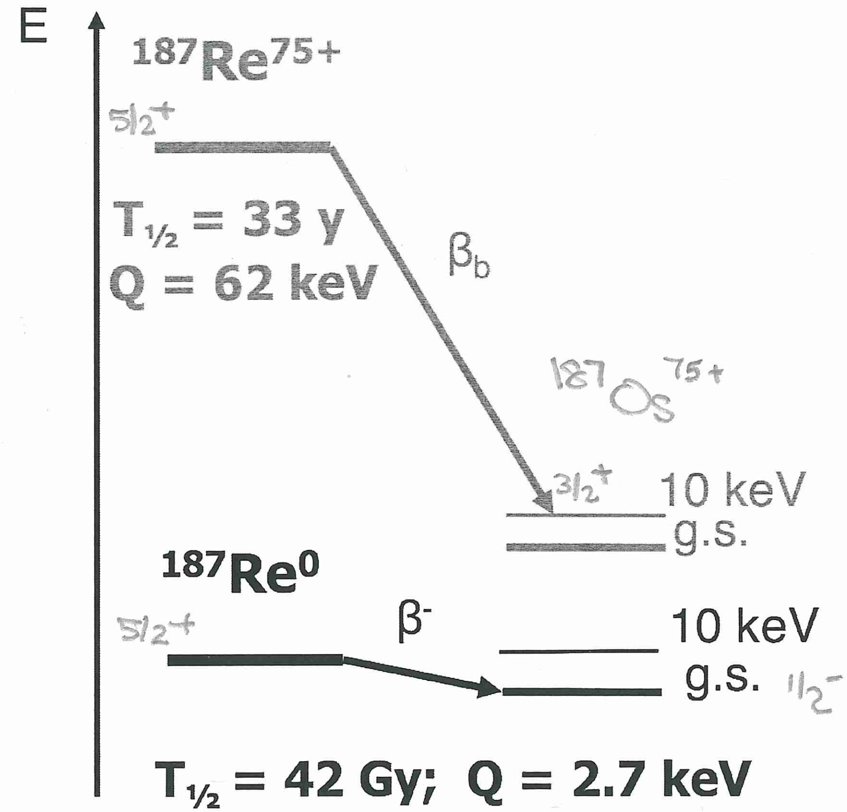
Bound-State β-decay



Bound-State β-decay of ^{187}Re



F. Bosch et al., Phys. Rev. Lett. 77 (1996) 5190

The 7 Nuclear Clocks for the Age of the Earth, the Solar System, the Galaxy, and the Universe

clock	$T_{1/2}[10^9$ y]
$^{40}K/^{40}Ar$ (□)	1.3
^{238}U…Th…^{206}Pb (□□)	4.5
^{232}Th…Ra…^{208}Pb (□□)	14
$^{176}Lu/^{176}Hf$ (□)	30
$^{187}Re/^{187}Os$ (□)	42
$^{87}Rb/^{87}Sr$ (□)	50
$^{147}Sm/^{143}Nd$ (□)	100

* Clayton (1964): *a mother-daughter couple ($^{187}Re/^{187}Os$) is the "best" radioactive clock*

This is but ONE complication

β - DECAY — more!

BOUND STATE β DECAY - see diagram

~24 NUCLEI potentially affected
- Boyd Table 3.3 & xerox

- NEED FULLY STRIPPED ION
  - 1st meas. ^{163}Dy (Jung et al. PRL 69, 2164, 1992
  - ^{187}Re (Bosch et al. PRL, 77, 5190, 1996)
  - REMOVAL OF e^- CHANGES THE BE & so the Q-value

		$\tau_{1/2}$
Q(Re atom)	= −7.1 keV	42 Gyr
Q(Re full ion)	= 2.64 keV	33 yrs

- Re-Os cosmochronology

- HEAVY EXAMPLES may affect r-s analyse
  - WHERE DISCUSSED?

NUCLEAR MODELS - simple ideas

- LIQUID DROP MODEL
- SHELL MODEL

} complimentary in a sense

- GOALS - fit measured properties of nuclei
  - provide bases for extrapolation to nuclei far beyond valley of stability, esp. for r-process

Table 3.4: Basic features of the principal theories on the atomic nucleus.

Liquid drop model	nuclear shell model
Nucleons equivalent: Protons and neutrons differ in charge, but all protons are equal, and all neutrons are equal	Nucleons differ not only by charge: Not a single nucleon is identical to another one present in a nucleus
Nucleons distributed homogeneously	Nucleons occupy discrete, quantum mechanical orbits
Electric charges distributed uniformly	Electric charges may be distributed nonuniformly
Final parameters: $\bar{E}_B$ as f(Z, N,(A)) Nucleon separation energies	Final parameters: $\bar{E}_B$ as f(quantum mechanics) Excitation energies Overall spin of the nucleus

LDM : A more critical than N or Z

: BUT n(Z,N) and properties → special cases for

Z = 2, 8, 20, 28, 50, 82

N = . - - - - - - . +126

This noted before theory

∴ called MAGIC

LDM and BINDING ENERGY

VOLUME

$$E_B = \alpha^{LDM} A - \beta^{LDM} A^{2/3} - \gamma^{LDM} \frac{Z^2}{A^{1/3}} - \zeta^{LDM} \frac{(N-Z)^2}{A} \pm \frac{\delta^{LDM}}{A^{3/4}} \quad (3.1)$$

SURFACE COULOMB ASYMMETRY PAIRING (+ / -)

α = 15.75 MeV

β = 15.94

γ = 0.665

ζ = 21.57

δ = 22.4

different sources → slightly different values, largely because of different sets of masses and E_B

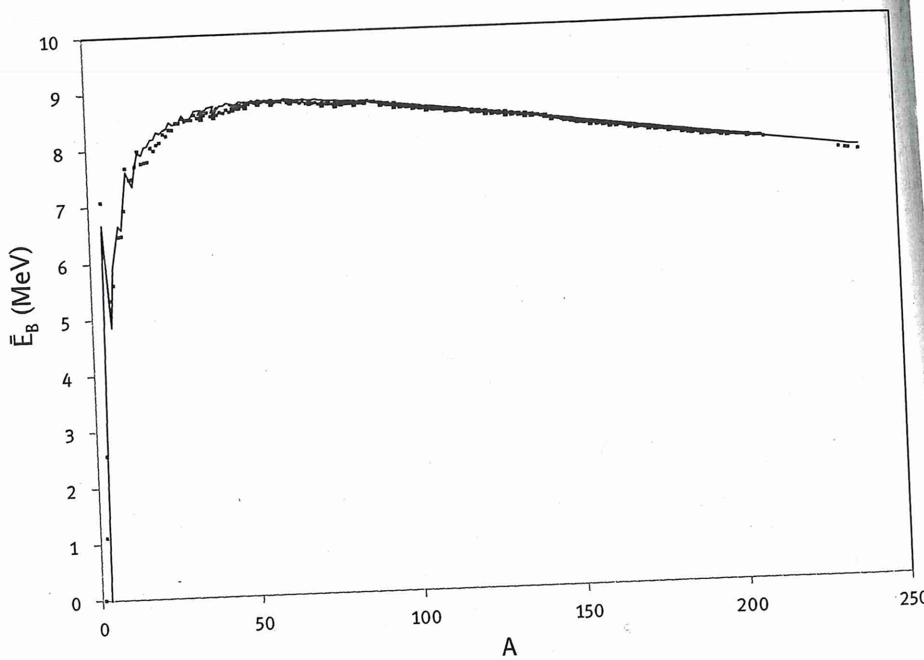


Figure 3.11: Mathematical outcome of the WEIZSÄCKER equation compared to "real" mean nucleon binding energies $\bar{E}_B$ versus mass number A for stable nuclei and the three uranium isotopes 234, 235 and 238 and ^{232}Th. Black squares are experimental values, the red line indicates the polynomial according to the WEIZSÄCKER equation.

- **VOLUME TERM** (+VE)

$\propto A = (N+Z)$

- **SURFACE TERM** (−VE) (

nucleons at surface less tightly bound

$S = 4\pi r^2 \propto A^{2/3}$

- **COULOMB TERM** (−ve)

$E_{coul} = \frac{3}{5} \frac{Z^2 e^2}{r}$ $r = r_0 A^{1/3}$

$= 0.72 \frac{Z^2}{A^{1/3}}$ but usually the coeff. is fitted.

- **ASYMMETRY TERM** (−ve)

- valley of stability favors $N > Z$ at high Z
- term $\frac{(N-Z)^2}{A}$ is introduced

• PAIRING TERM

• introduced to recognize fact that # stable nuclei depends on no. of Z and no. of N

Z, N	#	δ_{LDM}
even, even	166	$+\delta_{LDM}$
even, odd	55	0
odd, even	52	0
odd, odd	8	−1

$\frac{E_B}{A}$

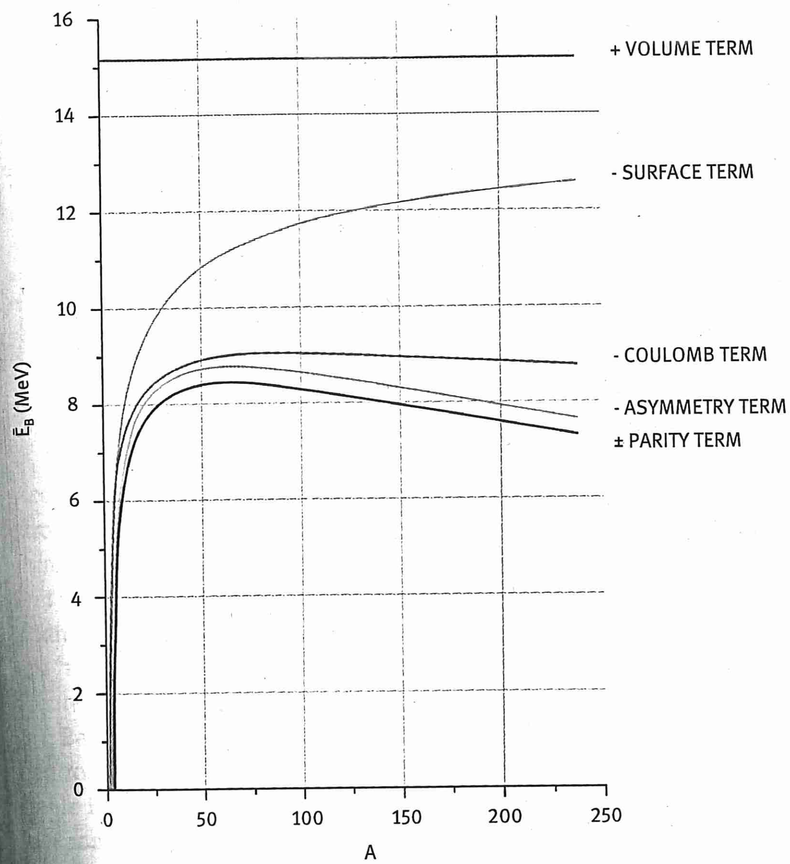


Figure 3.10: Output of the WEIZSÄCKER equation when successively using the five basic terms (shown in simplified and smoothed version).

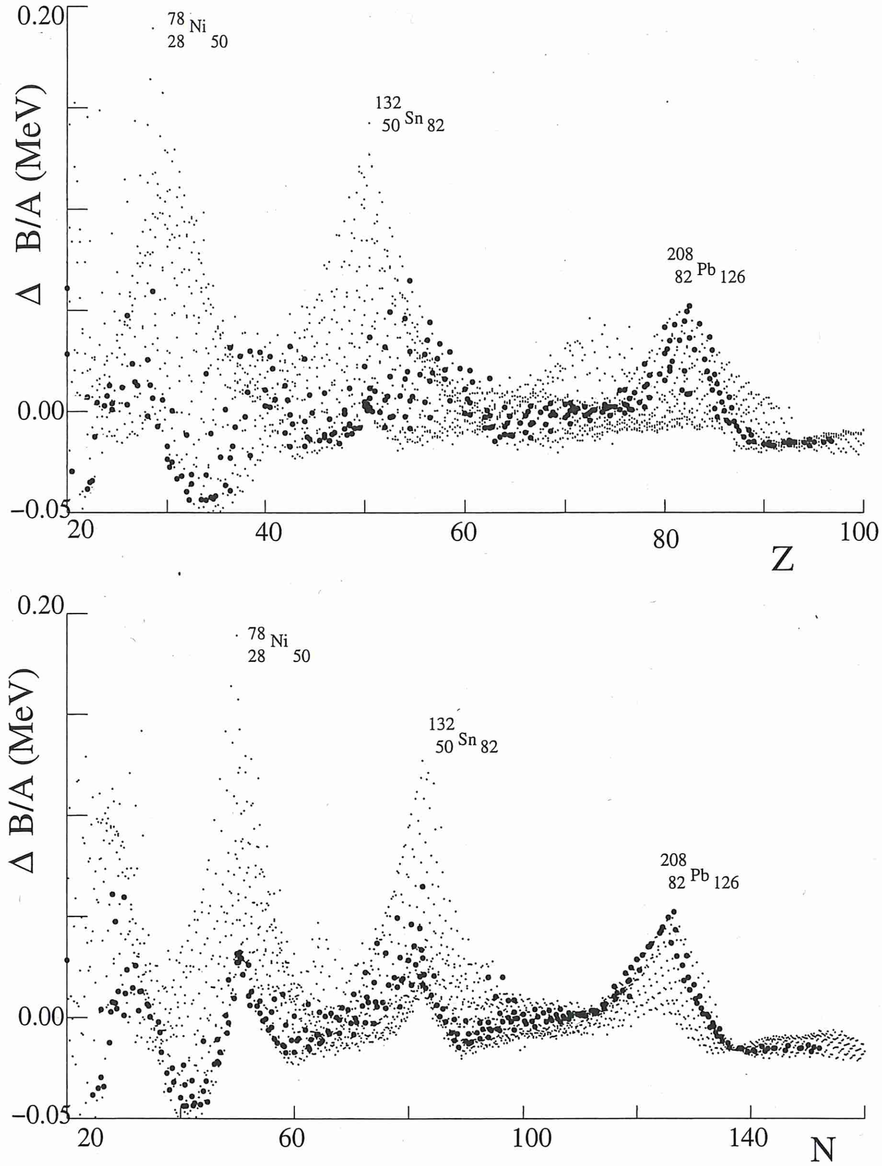


Fig. 2.8. Difference in MeV between the measured value of B/A and the value calculated with the empirical mass formula as a function of the number of protons Z (top) and of the number of neutrons N (bottom). The large dots are for β-stable nuclei. One can see maxima for the magic numbers $Z, N = 20$, 28, 50, 82, and 126. The largest excesses are for the doubly magic nuclides as indicated.

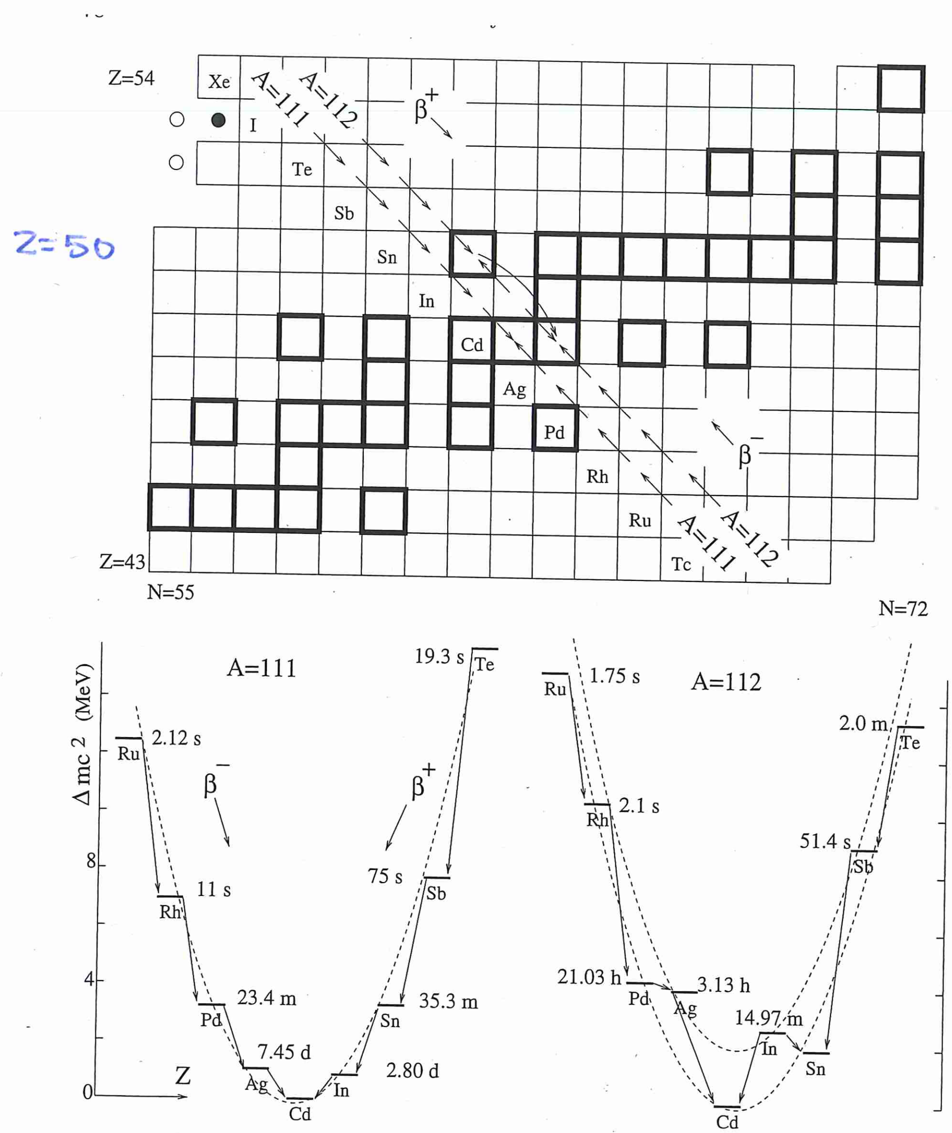


Fig. 2.6. The systematics of β-instability. The top panel shows a zoom of Fig. 2.1 with the β-stable nuclei shown with the heavy outlines. Nuclei with an excess of neutrons (below the β-stable nuclei) decay by β^- emission. Nuclei with an excess of protons (above the β-stable nuclei) decay by β^+ emission or electron capture. The bottom panel shows the atomic masses as a function of Z for $A = 111$ and $A = 112$. The quantity plotted is the difference between $m(Z)$ and the mass of the lightest isobar. The dashed lines show the predictions of the mass formula (2.13) after being offset so as to pass through the lowest mass isobars. Note that for even-A, there can be two β-stable isobars, e.g. ^{112}Sn and ^{112}Cd. The former decays by 2β-decay to the latter. The intermediate nucleus ^{112}In can decay to both.

NSM

IDEAL: DETERMINE NUCLEAR POTENTIAL and SOLVE S.E for energies etc. ab initio?

INTERIM: ADOPT MODEL POTENTIALS
- fit parameters → energies etc.

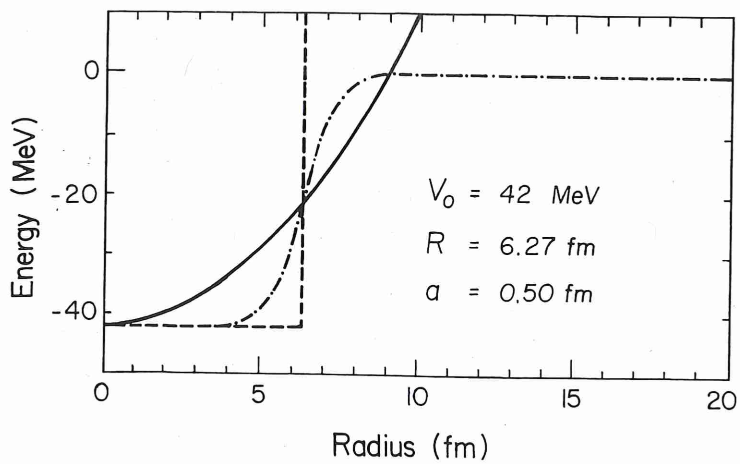


Fig. 2.1. Approximate potentials for the nuclear shell model. The solid curve represents the 3-dimensional harmonic oscillator potential, the dashed curve the infinite square well and the dot-dashed curve a more nearly realistic Woods–Saxon potential, $V(r) = -V_0/[1 + \exp\{(r - R)/a\}]$ (Woods & Saxon 1954). Adapted from Cowley (1995).

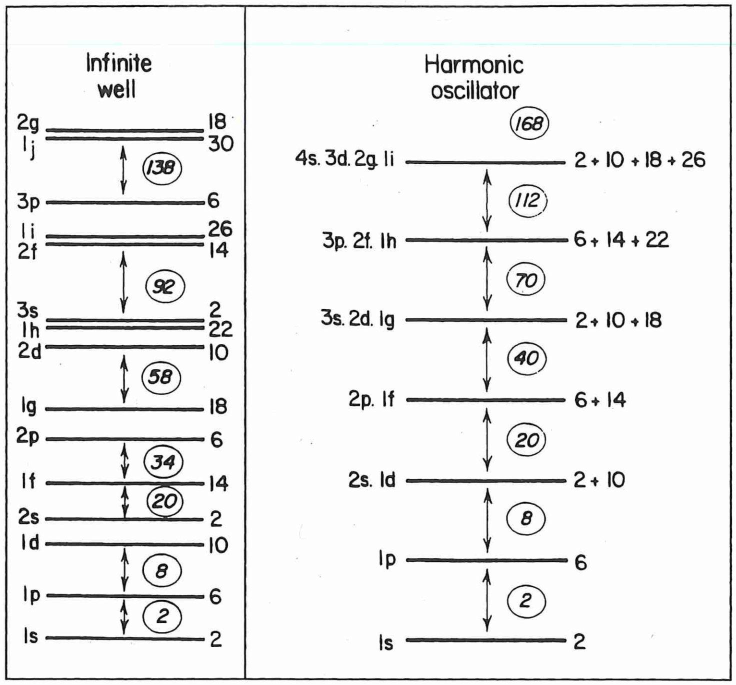


Fig. 2.2. Energy levels for ISW and 3DHO potentials. Each shows major gaps corresponding to closed shells and the numbers in circles give the cumulative number of protons or neutrons allowed by the Pauli principle. In a more realistic potential, the levels for given (n, ℓ) are intermediate between these extremes, in which the lower magic numbers 2, 8 and 20 are already apparent. Adapted from Krane (1987).

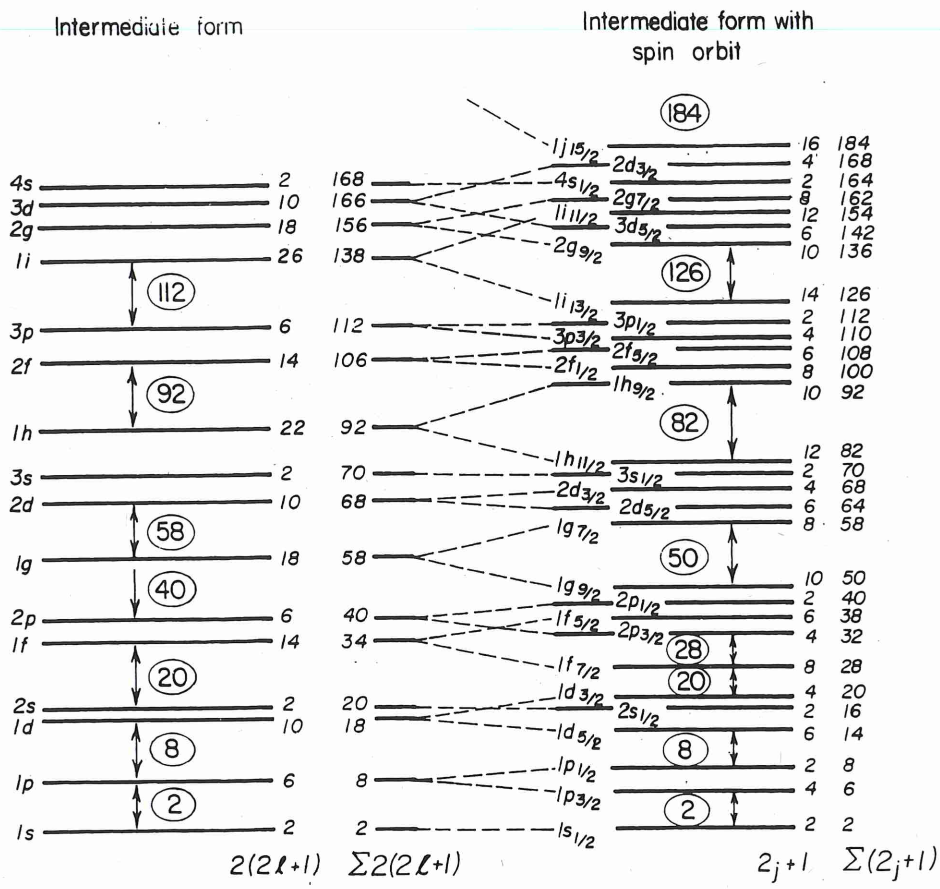


Fig. 2.3. At left, energy levels for a Woods–Saxon potential with $V_0 \simeq 50\,\text{MeV}$, $R = 1.25A^{1/3}\,\text{fm}$ and $a = 0.524\,\text{fm}$, neglecting spin–orbit interaction. At right, the same with spin–orbit term included. Adapted from Krane (1987).

ISW $E(n,l) \propto \pi^2\left(n+\frac{l}{2}\right)^2 - l(l+1)$

3HO $E(n,l) \propto \left(2n+l-\frac{1}{2}\right)$

Spin-orbit interaction $\propto \underline{l}\cdot\underline{s}$

$$\underline{j}^2 = (\underline{l}+\underline{s})^2 = \underline{l}^2 + 2\underline{l}\cdot\underline{s} + \underline{s}^2$$

$$\underline{l}\cdot\underline{s} = \frac{1}{2}\left(\underline{j}^2 - \underline{l}^2 - \underline{s}^2\right)$$

$$\langle \underline{l}\cdot\underline{s}\rangle = \frac{1}{2}\left[(j+1) - l(l+1) - s(s+1)\right]$$

and

$$\langle l\cdot s\rangle_{l+\frac{1}{2}} - \langle l\cdot s\rangle_{l-\frac{1}{2}} = \frac{1}{2}(2l+1)$$

$$\text{s-o} \propto -\frac{13}{A^{2/3}} \;[\text{MeV}]$$

This s-o interaction is not the same as that in atomic physics but tied "to quantum field properties of an assembly of neutrons".

NUCLEAR REACTIONS

- $A + a \rightarrow B + b + Q$ or $A(a, b)B$

  $Q > 0$ EXOTHERMIC

  $Q < 0$ ENDOTHERMIC

- Nucleosynthesis may involve
  A = nuclide with
  a = electrons
  protons, other nuclides
  photons
  neutrons
  neutrinos

CONSERVED QUANTITIES

- TOTAL ENERGY
- LINEAR MOMENTUM
- ELECTRIC CHARGE
- BARYON NUMBER
- LEPTON NUMBER
- PARITY (but weak interaction)
- SPIN
- ISOSPIN (not em interaction & some weak interactions)

TABLE 1

Reduced de Broglie wavelengths for various particles and energies

Energy (MeV)	Wavelength (fm)				
	Photon	Electron	Proton	α-Particle	^{16}O
0.1	2000.	589	14.5	7.2	3.61
1.	200	140	4.6	2.3	1.14
10	20.	18.7	1.4	0.72	0.36
100	2.0	2.0	0.45	0.23	0.11
1000	0.2	0.2	0.12	0.068	0.035

UNITS

1 eV ~ 10,000 K

1 barn = 10^{-24} cm^2

1 fm = 10^{-15} m

1 barn = 10^{+2} fm^2

The reduced de Broglie wavelength

$$\lambdabar = \frac{4.5}{\sqrt{m_{amu} E\,(\text{MeV})}} \quad \text{in fm}$$

CROSS-SECTIONS & RATE CONSTANTS

In general, $\sigma = \sigma(v)$

$$\text{Rate} = n_A n_B \int_0^\infty \phi(v)\,\sigma(v)\,v\,dv$$

$$= \frac{n_A n_B}{(1+\delta_{AB})} \langle \sigma v \rangle$$

Rate → $cm^{-3}\,s^{-1}$

$\langle \sigma v \rangle$ → $cm^3\,s^{-1}$

ASSUME MAXWELLIAN VELOCITY DIST'N

$$\phi(v)\,dv = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) dv$$

for non-degenerate gas

For A + B → C + D, velocity distribution needed is the <u>relative</u> velocity distribution

$m \rightarrow \mu$ = reduced mass

$$\frac{1}{\mu} = \frac{1}{m_A} + \frac{1}{m_B}$$

$$\mu = \frac{m_A m_B}{m_A + m_B}$$

$$\langle \sigma v \rangle = 4\pi \left(\frac{\mu}{2\pi kT} \right)^{3/2} \int_0^\infty v^3 \sigma(v) \exp\left(-\frac{\mu v^2}{2kT} \right) dv$$

$$= \left(\frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{kT} \right) dE$$

EXOTHERMIC $\sigma(E)$ = finite at $E = 0$

ENDOTHERMIC $\sigma(E) = 0$ for $E \leq -Q$