Comments on Assignment 5

Preamble on nuclear reaction rates

For the reaction $1 + 2 \rightarrow 3 + 4$ with $1 \neq 2$,

$$\frac{dn_1}{dt} = -n_1 n_2 < \sigma v >_{12} \tag{1}$$

where the rate constant $< \sigma v >_{12}$ has the units $cm^{-3} sec^{-1}$.

Given the density, ρ (in g/cm^{-3}), and mass fractions $X_1 = \rho_1/\rho$ with $n_1 = \rho_1/(A_1 M_u)$ where A_1 = atomic mass of 1 and M_u is the atomic mass unit, $M_u = 1.6605 \times 10^{-24}\, g$ and introducing Avagadro's number $N_A = M_u^{-1} = 6.02214 \times 10^{23}$, we may write (1) as

$$\begin{aligned}\frac{dn_1}{dt} &= \rho_2 N_A^2 (\frac{X_1}{A_1})(\frac{X_2}{A_2}) < \sigma v > \\ &\equiv \rho_2 N_A (\frac{X_1}{A_1})(\frac{X_2}{A_2})[\sigma v]\end{aligned}$$

where $[\sigma v] = N_A < \sigma v >$ with units $cm^{-3} sec^{-1} mol^{-1}$.

The mean lifetime of particle 1 is

$$\begin{aligned}\tau = \frac{1}{n_1}\frac{dn_1}{dt} &= (\frac{1}{X_1})(\frac{dX_1}{dt}) = n_2 < \sigma v >_{12} \\ &= \rho N_A (\frac{X_2}{A_2}) < \sigma v >_{12} \\ &= \rho (\frac{X_2}{A_2})[\sigma v]_{12}\end{aligned}$$

where $[\sigma v]_{12} = N_A < \sigma v >_{12}$ with units $cm^{-3} sec^{-1} mol^{-1}$.

Now, consider the case of ^{13}N in the 10 solar mass core, where

$$T_9 = 0.031\, K \text{ and } \rho = 8.9\, g/cm^{-3}$$

and ASSUME for H that $H/A_H = 0.7$ which will be true at the start of H-burning in the core.

The NACRE II rate for $T_9 = 0.030$ is $3.84 \times 10^{-13}\, cm^{-3} sec^{-1} mol^{-1}$

Then, τ_{13} for $^{13}N(\rho, \gamma)^{14}O$ is

$$\begin{aligned}\tau_{13} &= 8.9 \times 0.7 \times (3.84 \times 10^{-13}) \\ &= 2.4 \times 10^{-12}\end{aligned}$$

i.e., VERY short relative to the $\beta-$decay half-life of 9.97 minutes.

As a check, let us use Iliadis eqn. (5.149) on page 464 and the corresponding figure (5.48).

The (ρ, T) just considered are off the figure and way below the (solid) line corresponding to $\tau_p(^{13}N) = \tau_\beta(^{13}N)$.

Now, we are finding the density at which the lifetimes for proton capture and $\beta-$ decay are equal,

$$\begin{aligned}\rho &= \frac{\ln 2}{T_{1/2}} \frac{1}{\frac{X_H}{M_H}[N_A < \sigma v >]} \\ &= \frac{0.693}{598} \frac{1}{0.7(1.41 \times 10^{-5})} = 117\, g/cm^3\end{aligned}$$

Figure 5.48 gives $\rho \simeq 100\, gcm^3$. (I suppose the difference may be due to different adopted rates for $< \sigma v >$).

Use of $[\sigma v]$ in $cm^{-3} sec^{-1} mol^{-1}$ may have been introduced by Fowler, Caughbu & Zimmerman (1967, ARAA). Iliadis slips the notation in on page 14 and almost everywhere explicitly retains N_A, as in Figure 3.29 where the y-axis is labeled $N_A < \sigma v >$ in units of $cm^{-3} sec^{-1} mol^{-1}$.

2. Operation of CNO1 is discussed by Iliadis. See equation (5.57)-(5.60) and Figure 5.11.

   From the point of view of nucleosynthesis:

   - $^{12}C/^{13}C \sim 4$ with a weak T dependence;
   - $^{12}C/^{14}N \ll 1$ with a strong T dependence and conversion of 'all' C to N;
   - $^{14}N/^{15}N \gg 1$ with a weak T dependence;

3. At one solar mass, the pp-chains are the dominant mode for conversion of H to He. Apart

   from H and ^4He, the sole illustrated nuclide involved in the pp-chain is ^3He. Iliadis discusses synthesis and astration of ^3He on pages 359-361, see Figure 5.4.

In the outer layers, say $M(R) > 0.7$, synthesis of ^3He increases with increasing temperature. See Figure 5.4(b) which shows that the time to reach the equilibrium abundance increases sharply as temperature is lowered. Thus ^3He outside $M(R) > 0.7$ does not reach its equilibrium abundance.

As its peak abundance at $M(R) \sim 0.65$, ^3He may be close to its equilibrium abundance. The table attached to the assignment gives $\log T = 6.793$ at $M(R) = 0.65$ or $T = 0.0062\, GK$. Fig. 5.4(b) says achieving equilibrium takes 10^{10} yrs at $\rho \simeq 100\, g/cm^3$ and $X_H = 0.5$. Actual density is less then this and, therefore, time to achieve equilibrium will be longer (see eqn. 5.20) (Note too that Fig. 5.4 gives $^3He/H$ but the Dearborn figure gives the mass fraction of ^3He.) In brief, the ^3He peak at $M(R) \sim 0.65$ will be somewhat less than the equilibrium value for $T = 0.0062\, GK$.

Interior to $M(R) \sim 0.65$, ^3He is likely close to its equilibrium value - see Fig. 5.4(a) and notably Fig. 5.4(b) for the shorter times to equilibrium at higher temperatures. At solar center

$T = 0.015\, GK$, and it takes $\sim 10^5\, yrs$ to reach ^3He equilibrium.

Synthesis of ^3He in low mass stars is very relevant to extracting Big Bang production of ^3He from $^3He/H$ observations of Galactic HII regions. Has synthesis of ^3He in low mass stars and subsequent mass loss enriched the Galaxy in ^3He? I do not think there s a definitive answer but the fact that the observed $^3He/H$ ratios are independent of Galactocentric distance suggests low mass stars have not contributed much. Observations of 'high' $^3He/H$ ratios for planetary nebulae shows that ^3He can be ejected. See atro-ph 1604.02679.

Partial operation for He CNO I cycle accounts for the CNO changes.

4. See above discussion on ^3He and Iliadis.