AST 353 Astrophysics — Problem Set 3

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I. GROWTH OF PERTURBATIONS

Equation (1) in the question governs the growth of overdensity in the universe at early times, when $\delta \ll 1$. In this question we wish to numerically solve it. It is a second-order ODE in $\delta(t)$, with the functions $H(t)$ and $\bar{\rho}(t)$ known from PS 1.

We have solved the equation analytically for the simple case of Einstein-de Sitter (EdS) background universe (see Lecture 8 notes). The solution, keeping only the growing mode, reads

$$\delta_{\rm EdS}(t) = At^{2/3}, \tag{1}$$

where A is a constant. With the given initial condition of $\delta_i = 10^{-4}$ at initial time t_i, we can renormalize the equation into

$$\delta_{\rm EdS} = \delta_i \left(\frac{t}{t_i}\right)^{2/3} \tag{2}$$

We need to provide the numerical solver ('NDSolve' in Mathematica) the following:

1. the equation itselt;
2. all known functions: $H(t)$ and $\bar{\rho}(t)$;
3. two intial conditions (because the ODE is of second order).

In PS 1, we have already gathered the fuctions H and ρ, but they are function of z:

$$H(z) = \frac{\dot{a}}{a} = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda} \tag{3}$$

$$\bar{\rho}(z) = \frac{3H^2(z)}{8\pi G} \tag{4}$$

We can transform them into functions of t simply by constructing a function z(t), such that

$$H(t) = H(z(t)) \tag{5}$$

$$\bar{\rho}(t) = \rho(\bar{z}(t)). \tag{6}$$

Recall from PS 1, we numerically integrated

$$t(z) = \frac{1}{H_0}\int_z^\infty \frac{dz'}{(1+z')\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}. \tag{7}$$

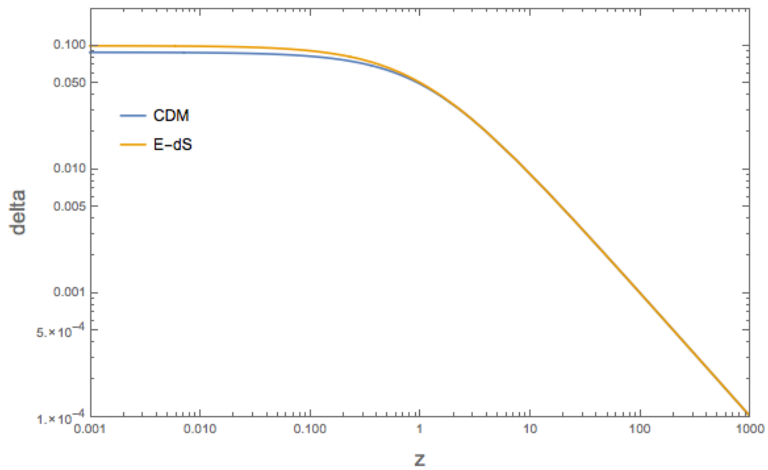
We can numerically construct a 'lookup table' for z given t. This can be done using the "Interpolation" routine in Mathematica.

For the initial conditions, we have

$$\delta(t(z_i)) = \delta_i = 10^{-4} \tag{8}$$

$$\dot{\delta}(t(z_i)) = \dot{\delta}_{\rm EdS}(t(z_i)) \tag{9}$$

Using the "NDSolve" equation solver in Mathematica, we finally obtain



II. BARYONIC COLLAPSE

This question concerns with the conditions for the collapse of gas onto the DM halo, see Lecture 12 notes for details. The condition for the baryonic collapse is

$$T_{vir} = 10^4\,\mathrm{K}\left(\frac{M_{halo}}{10^8 M_\odot}\right)^{2/3}\left(\frac{1+z_{vir}}{10}\right) \tag{10}$$

Recall by definition T_{vir} represents the maximum temperature the gas cloud can have for gravity of the DM halo to be strong enough to induce a collapse. Substituting $T_{vir} = 5000\,\mathrm{K}$ and $z_{vir} = 10$, we have

$$M_{halo} \simeq 3.1 \times 10^7\, M_\odot \tag{11}$$