AST353 (Spring 2016)

ASTROPHYSICS

Problem Set 5

Due in class: Thursday, April 21, 2016

(worth 10/100)

1. Simple stellar model

Assume a star has a radius of $R = 3R_\odot$ and a quadratic density profile:

$$\rho(r) = \rho_c \left[1 - \left(\frac{r}{R}\right)^2\right] \quad .$$

Here, $\rho_c = 20$ g cm^{-3} is the central density.

a. What is the mass M of the star (in units of the solar mass $M_\odot$)?

b. What is the average density, $<\rho>$, for this star? What is the free-fall time, $t_{\rm ff}$ (pick suitable units here!)? Very briefly explain the physical meaning of $t_{\rm ff}$!

c. Solve the equation of hydrostatic equilibrium to find the pressure, $P(r)$, as a function of radius. When doing the integration, assume that the pressure drops to zero at the outer boundary, $P(R) = 0$ (so-called *zero boundary condition*). Express your answer in the form:

$$P(r) = K\left[1 + a_1 x + a_2 x^2 + a_3 x^3 + ...\right] \quad ,$$

where $x = r/R$. Here, $a_1, a_2, a_3, ...$ are numerical constants, and your job is to find them. Also, you need to determine the constant K in front of the polynomial.

d. Find an expression for the central pressure P_c, and evaluate it for this star (in units of dyn cm^{-2})! Compare this with the value you get using the approximate formula for P_c that we have derived in class!

e. Find an expression for the total gravitational potential energy, $E_{\rm pot}$, of such a star! Express your result first in a general form, as a function of stellar mass and radius. You should find $E_{\rm pot} = -$ const GM^2/R, and your job is to determine the numerical constant. Then evaluate this expression for the particular star in this problem (in units of erg)!