AST353 (Spring 2016)

ASTROPHYSICS

Problem Set 3

Due in class: Thursday, March 3, 2016

(worth 10/100)

1. Growth of Perturbations

In class, we derived the equation for the evolution of the overdensity, valid as long as $\delta \ll 1$:

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\bar{\rho}\delta \ . \tag{1}$$

In class, we solved this equation analytically for the simple case of an Einstein-de Sitter (E-dS) background universe (see our lecture notes).

a. Now, (numerically) work out the solution of this equation, $\delta(t)$ or $\delta(z)$, for the case of the more complicated background universe that we actually live in. Recall that you had worked out the evolution of our universe in PS1, part 1, and you should use the results that you got there (use the same cosmological parameters given there). As in PS1, you may wish to use MATHEMATICA to do the numerical integration.

As initial conditions, pick an overdensity with $\delta_i = 10^{-4}$ at $z_i = 1,000$. Solve Equation (1) from z_i to $z = 0$.

b. Plot the result with δ on the y-axis, and redshift, z, on the x-axis. Experiment a bit with the plotting, to make this look nice; e.g., find out what kind of scaling, and range for the axes is good.

c. Overplot, with a different line style, on the same plot as for part b, the E-dS analytical solution, using the same initial conditions.

Why do the two curves diverge at lower redshifts (later times)?

2. Baryonic Collapse

Consider gas of the intergalactic medium (IGM) at a redshift of $z = 10$. For simplicity, assume that the gas is pure, neutral, hydrogen, and has a temperature of $T = 5,000\,\mathrm{K}$ (say, because it was heated to this temperature somehow).

What is the minimum mass (in units of solar mass) that a (virialized) dark-matter halo needs to have to be able to attract the diffuse IGM gas and induce it to collapse into the halo?