Hints on Group Project.

① Harrison - Zeldovich power spectrum $P_{HZ}(k)$ is "scale free".

It means something does NOT depend on the wavenumber k, or the spatial scale. What is that?

② We would like to evaluate and plot the $z=0$ line in Loeb's Figure 3.1.

$$\sigma^2(M) \equiv \sigma^2(R) = \int_0^\infty \frac{dk}{2\pi^2} \cdot k^2 \cdot P(k) \left(\frac{3 j_1(kR)}{kR} \right)^2.$$

Relating M and R:

$$\begin{cases} M = \frac{4}{3}\pi R^3 \cdot \rho_m \\ \rho_m = \Omega_m \rho_0 . \end{cases}$$

j_1 : spherical Bessel function of the first kind, $j_1(x) = (\sin x - x\cos x)/x^2$.

② (Continued) 03-07-2016

- The primordial power spectrum

$$P_{pri}(k) \propto k^{n_s}, \text{ where } n_s \approx 1.$$

Shape:

- The actual $P(k)$ we have to integrate is modified by the transfer function $T^2(k)$; we can approximate $T^2(k)$ analytically by

$$P(k) \propto T^2(k) \cdot P_{pri}(k) \propto k^{n_s} / (1 + \alpha_p k + \beta_p k^2)^2,$$

with $\alpha_p = 8(\Omega_m h^2)^{-1}$ and

$\beta_p = 4.7 (\Omega_m h^2)^{-2}$,

$h = 0.7$.

$\therefore$ $P(k) \propto k^1$, small k.

$P(k) \propto k^{-3}$, large k

Normalization:

- After fixing the shape of the power spectrum $P(k)$, we need to properly normalize the $\sigma(M)$ plot.

Normalization: (continued) 03-07-2016

- We don't know inflation enough to work out the normalization from first principles, we can however do it by matching the $\sigma(M)$ with observation.

- From the measurements of CMB, we obtain the normalization in terms of the parameter σ_8, which is the σ value at $8\,h^{-1}\,\mathrm{Mpc}$ scale.*

$$\sigma(R = 8h^{-1}\mathrm{Mpc}) \equiv \sigma_8 = 0.82 \approx 1.0.$$

- With the above shape & normalization, we should be able to obtain the $\sigma(M)$ plot. Remember our aim is not to reproduce it exactly, just try to get the shape close.

*Note: scale of $R = 8\,h^{-1}\,\mathrm{Mpc}$ is about the size of galaxy clusters.