In previous lectures, we answered the first two questions in structure formation:

① what are the initial fluctuations?

and ② How do the perturbations grow?

Note: Growth in

- static medium : exponential
  $\delta \propto \exp(-t/t_{ff})$

  → Initial conditions don't matter!

- Expanding medium : power-law
  $\delta \propto t^{2/3}$ (for flat universe).

  → Initial conditions matter !!

In this lecture, we will start answering the last questions we posted about structure formation —

③ what happens when $\delta \gtrsim 1$?

→ "halo" formation

OR, → "Virialization".

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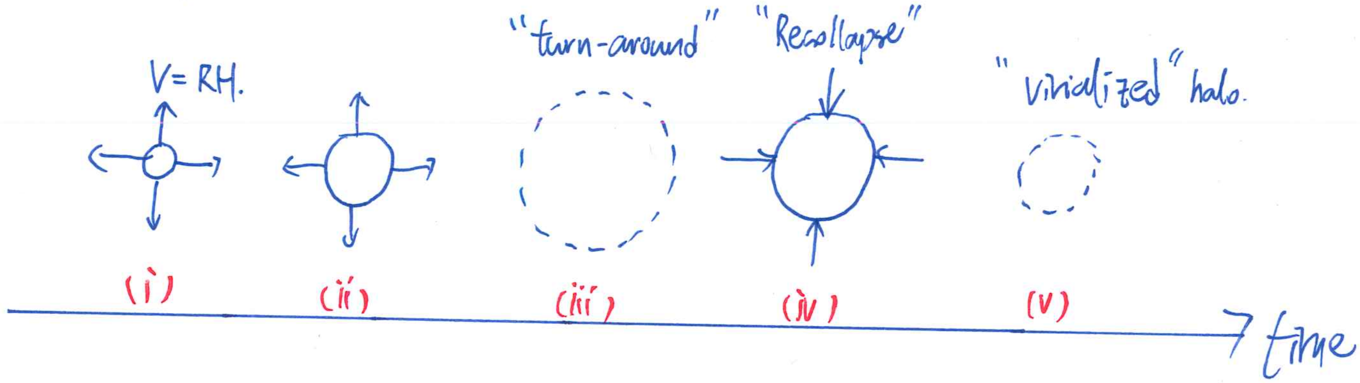
First, let's look at a cartoon picture of the virialization process:



(i). Initially, the overdense region expands with the background universe according to the Hubble's law.

(ii). The expansion slows down, "detaching" from the background universe.

(iii). At "turn-around", the overdensity stops expanding.

(iv). It starts to recollapse under its own gravity

(V). "Virialization" —— the recollapsing dark matter particles miss the central point ($R=0 \rightarrow$ "singularity") because of random motion.

$\rightarrow$ establish the state of "Virial Equilibrium" V.E.

where $\boxed{2 E_{kin} \simeq - E_{pot}}$ "Virial Theorem".

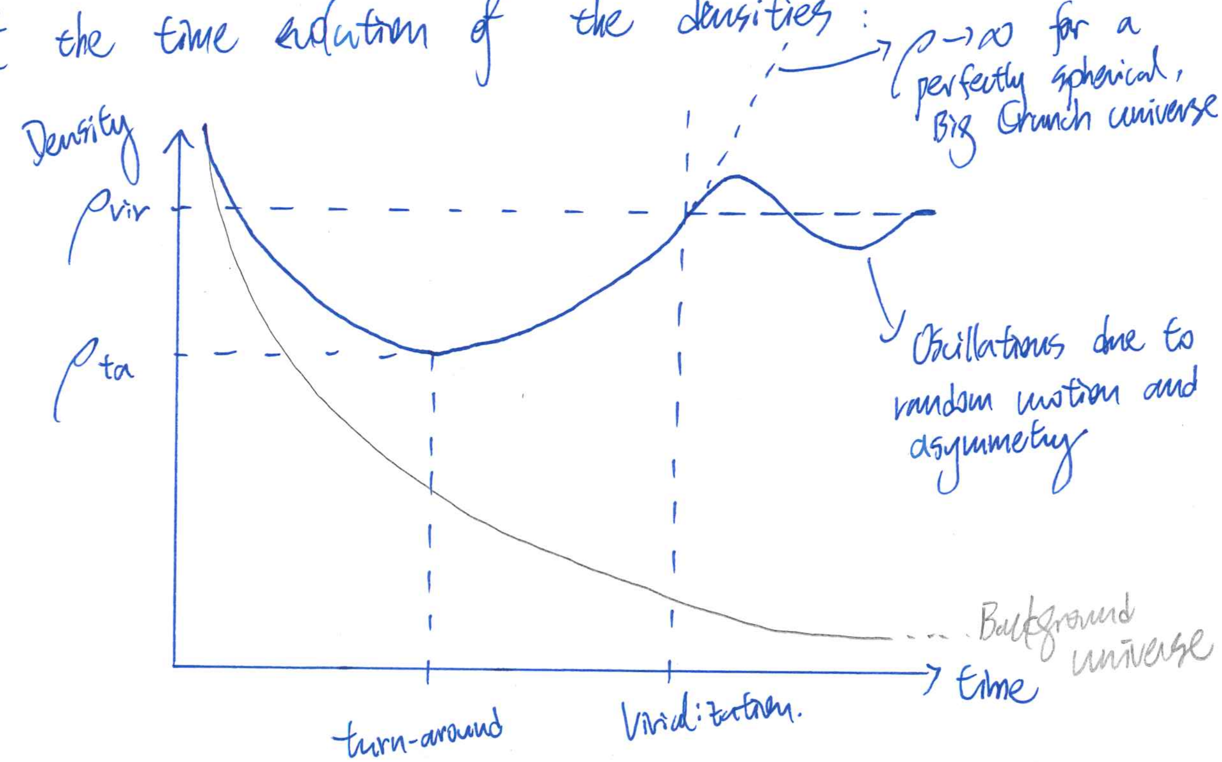
Q: why is Virial Theorem so important in astrophysics?

A: For spherical system, VT reads:

$$v^2 = \frac{GM}{R}.$$

It helps us to get order of magnitude estimation of observed systems as the quantities are connected to observables.

Let's look at the time evolution of the densities:



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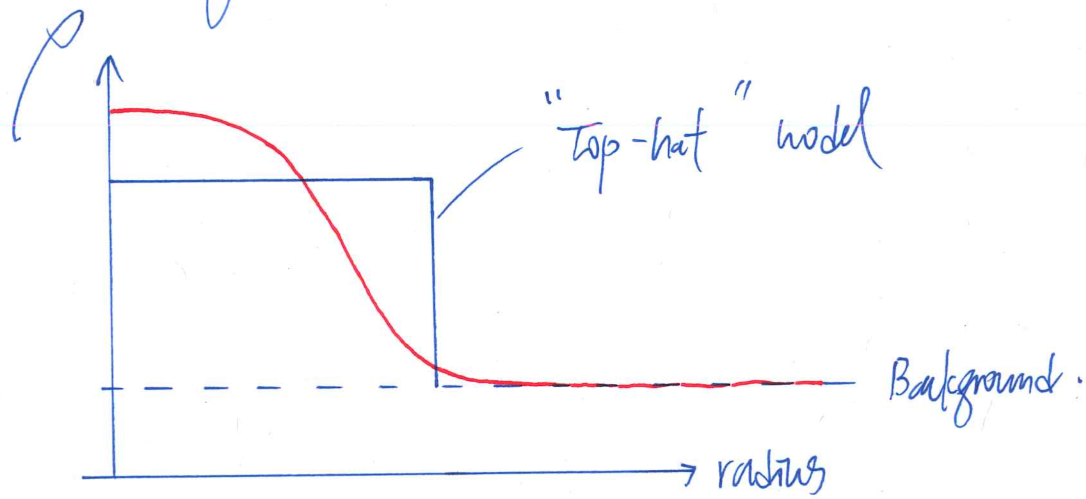
Note: radial density profile in overdensity



- Idea: embed mini-Big Crunch (PS1, #2) into simple background universe.

  ↳ Einstein-de-Sitter [EdS] model.

Review: Basics of E-dS
→ flat (non-curved) universe with only matter.

$$\rho_{tot} = \rho_m$$

$$E_{tot} = E_{kin} + E_{pot} = 0.$$

Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{tot} \qquad (k=0)$$

$$\rho_{tot} = \rho_m = \rho_{m,0}\, a^{-3}$$

$$a \propto t^{2/3}$$

$$\Rightarrow \quad H(t) = \frac{\dot{a}}{a} = \frac{2}{3t}$$

$$\bar{\rho}(t) = \frac{3H^2(t)}{8\pi G} = \frac{1}{6\pi G t^2}.$$

We will continue to derive ρ_{vir} in the next lecture.