AST 353 Astrophysics. 02-09-2016

Recall we raised three questions at the end of last lecture regarding structure formation. Today we will approach the first one:

① What are the initial fluctuations?

[Note: the math in the following is not hard, but the concepts are pretty advanced. Make sure to pause and really think about it, let me know if you find anything unclear.]

- Construct a spherical 'window' with radius R.

$$M = \frac{4}{3}\pi R^3 \cdot \bar{\rho}$$

Let's say the mass within this window is M, the average density of the universe is $\bar{\rho}$.

- Now place this window everywhere in the universe, then calculate ρ, then $\delta_M = \frac{\rho - \bar{\rho}}{\bar{\rho}}$

- Next, calculate the average value of δ_M obtained by putting the window everywhere. You will find

$$\langle \delta_M \rangle = 0$$

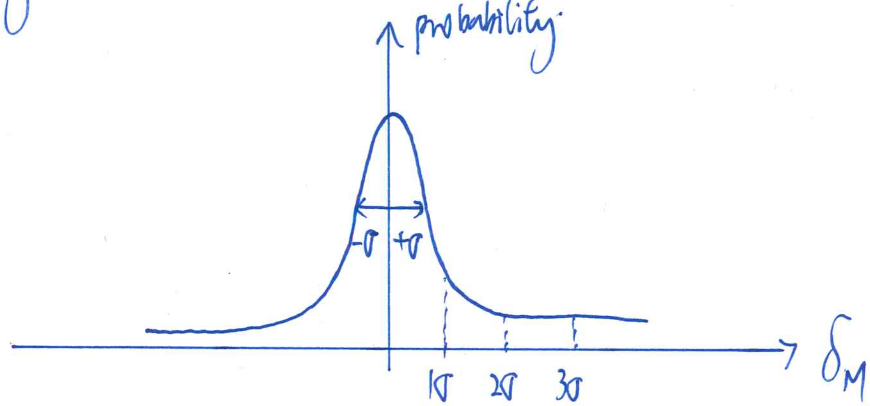
This can be understood when you consider any over-density, there is an underdensity so they average out.

- Then, if we take the square before averaging: 02-09-2016

define: $\langle \delta_M^2 \rangle \equiv \sigma^2(M)$

This is analogous to the variance, which measures how "spread out" the measured δ_M is.

Here is a little recap of probability distribution. The most commonly seen probability distribution is the Gaussian distribution.



Mathematically,

$$P(\delta_M) = \frac{1}{\sqrt{2\pi\sigma_M^2}} \cdot e^{-\delta_M^2/2\sigma_M^2}$$

Define:

1 - sigma	68%
2 - sigma	95.5%
3 - sigma	99.7%.

what it means is that if you integrate the above probability distribution function $P(\delta_M)$ from $-\sigma$ to $+\sigma$, you will get 0.68 (out of total probability of 1.)

· Remember we picked / set a mass scale M for the spherical window, so we will have a distribution curve like the above for every mass scale. 02-09-2016

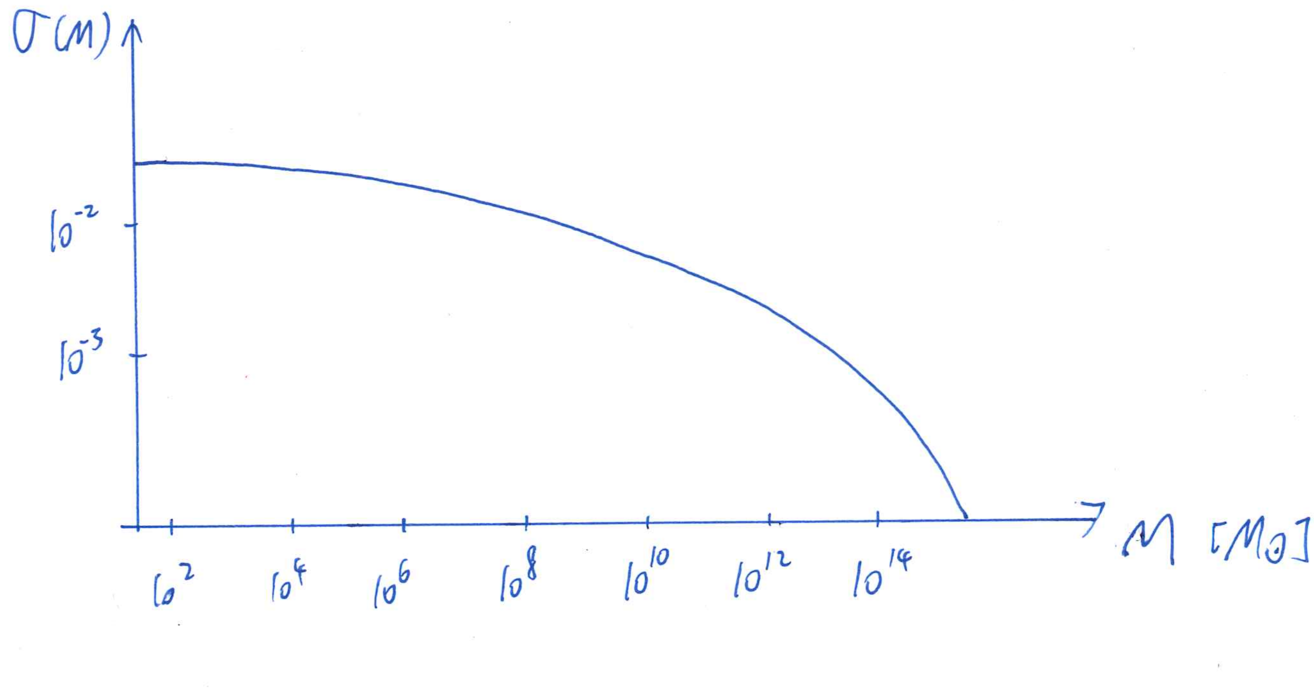
· If we wish to "measure" the fluctuations at a given time in the cosmic history, we would need to pick a time.

→ we pick the moment after "recombination".
It was the moment when the primordial fireball universe was cooled down enough for protons and electrons to combine and form neutral hydrogen.

$z_{recomb.} \sim 1000$

$t_{recomb} \sim 0.5\,Myr$

The fluctuations at different mass scales look like this:



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- As time goes by gravity will amplify the fluctuation. "Lifting" the curve to higher and higher σ values.

Recall we roughly need $\delta \sim 1$ for structure formation.

This implies we have a

"bottom-up structure formation", where structures with small mass scales form first.

we commonly refer to this model as the

ΛCDM model.

→ We will expand more on this in Group Project 1!

The next part of the lecture continued on the ppt slides.
