Last time we derived the Friedmann Equation using Newtonian cosmology, which reads

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho_{tot} - \frac{kc^2}{a^2}.$$

It turns out we will get the same equation if we applied GR in the derivation, with the k term representing the space geometry. In our derivation, the k term is simply an integration constant.

In this lecture we continue our brief intro. to cosmology (part 3 of 3).

Recall

$$\rho_m(t) = \rho_{m,0} \cdot \frac{1}{a^3(t)} = \rho_{m,0}(1+z)^3$$

$$\rho_\Lambda(t) = \rho_{\Lambda,0} = \text{constant}.$$

$$\Rightarrow \quad \rho_{tot}(t) = \rho_m(t) + \rho_\Lambda(t).$$

We also defined the density parameters:

$$\Omega_m = \rho_{m,0} / \rho_{tot,0}$$

$$\Omega_\Lambda = \rho_{\Lambda,0} / \rho_{tot,0}.$$

From observation we found $\Omega_m \simeq 0.30$; $\Omega_\Lambda \simeq 0.70$

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$$H_0^2 = H^2(t_0) = \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{8\pi G}{3}\cdot \rho_{tot,0}$$

$$\Rightarrow \quad H_0^2 = \frac{8\pi G}{3}\cdot \rho_{tot,0}$$

$$\Rightarrow \quad \rho_{tot,0} = \frac{3H_0^2}{8\pi G}$$

$$H_0 = 70 \text{ km } s^{-1} \text{ Mpc}^{-1}.$$

$$\Rightarrow \quad \frac{\dot{a}}{a} = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}$$

$\rightarrow$ Solve for $t(z)$.

$$a = (1+z)^{-1}$$

$$\Rightarrow \quad \dot{a} = -\frac{1}{(1+z)^2}\cdot\frac{dz}{dt}$$

$$\Rightarrow \quad \frac{\dot{a}}{a} = -\frac{1}{(1+z)}\cdot\frac{dz}{dt} = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}$$

Separation of variables:

$$\int_0^{t(z)} dt' = \frac{1}{H_0}\cdot\int_\infty^z \frac{dz'}{(1+z')\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}$$

$$\Rightarrow \quad t(z) = \frac{1}{H_0}\cdot\int_z^\infty \frac{dz'}{(1+z')\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}}$$

Schematically: