AST 353 Astrophysics. 01-26-2016.

Intro. to Cosmology (continued).

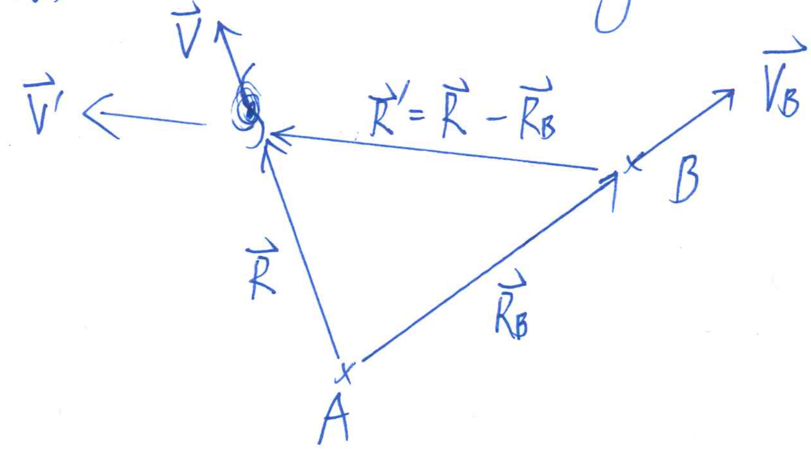
Last time we ended by introducing the Hubble's law, which reads

$$\vec{V} = H_0 \cdot \vec{R} .$$

Vector signs have been added to the law to emphasize that it is a vector law, which applies to our 3D universe.

In a universe where the Hubble's law applies, there is <u>NO</u> special / privileged observer, let's see how.

Consider two observers A and B, observing the same galaxy:



what we observe: $\vec{V} = H_0 \vec{R}$ and $\vec{V_B} = H_0 \vec{R_B}$

observer B sees: $\vec{V'} = \vec{V} - \vec{R_B} = H_0(\vec{R} - \vec{R_B}) = H_0 \vec{R'}$.

$\boxed{\vec{V'} = H_0 \vec{R'}}$ $\Longrightarrow$ observer B sees the same law!!

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Note: this universal expansion only works for linear relations.

Hubble law $\longrightarrow$ uniform expansion.

$\begin{cases} \text{No special location} \\ \text{No special direction.} \end{cases}$

Einstein : the "Cosmological principle".

Then, we introduced the powerful 'sloppy calculus':

$$H(t) = \frac{\dot{a}}{a}$$

At present day,

$$H_0 = H(t_0) = \frac{\dot{a}_0}{a_0} = \dot{a}_0 = \frac{da_0}{dt}$$

$$\sim \frac{a_0}{t_0} = \frac{1}{t_0}$$

$\Rightarrow \quad t_0 \sim \frac{1}{H_0}$ (expression of Hubble time).

· Cosmic dynamics. 01-26-2016

To describe the expansion of the universe, we need the "equation of motion" of the universe.

→ strictly, we should use Einstein's General Relativity (GR).

→ Here, we will use a short-cut, Newtonian Cosmology.

The Newtonian picture of gravity is of course incomplete. But the question we should be asking here is:

Q: under what conditions are we required to consider GR in cosmology?

A: For GR we must compare the size of an object R with its Schwarzschild radius R_S. Formally,

$$R_S \equiv \frac{2GM}{c^2}.$$

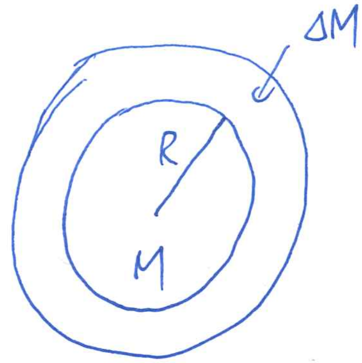
If $R \gg R_S$, the gravity is weak and GR is not needed.

Take for example our Sun,

$$\frac{R_{S,\odot}}{R_\odot} \sim \frac{3\,\text{km}}{10^6\,\text{km}} \sim 10^{-6} \Rightarrow$$ Small but measurable GR effect.

Again, GR is indeed necessary in cosmology. But in here we will try to build a Newtonian model. 01-26-2016

Consider a sphere of mass M with radius R, and an additional shell of mass ΔM.



ρ: mean mass density.

$M = \frac{4\pi}{3} R^3 \cdot \rho$

Newton's law $\qquad F_G = \Delta M \cdot \ddot{R}$

$$\Rightarrow \quad -\frac{G \cdot M \cdot \Delta M}{R^2} = \Delta M \cdot \ddot{R}$$

Integrate once over time, using the tricks

(1) Multiply both sides by $2\dot{R}$

(2) Use $\frac{d}{dt}(\dot{R})^2 = 2\dot{R}\ddot{R}$

$$\Rightarrow \int \frac{d}{dt}(\dot{R})^2 dt = -2GM \int \frac{dR}{dt} \cdot R^{-2} \cdot dt$$

$$\Rightarrow \int d(\dot{R})^2 = -2GM \int R^{-2} dR$$

$$\Rightarrow (\dot{R})^2 = \frac{2GM}{R} + \text{constant}.$$

We then label the constant of integration as 01-26-2016.
" $-kc^2$ ", where k is a dimensionless number and c is the speed of light. We do this by considering the units of the other terms in the equation, e.g. $(\dot{R})^2$.

$$\Rightarrow \quad \dot{R}^2 = \frac{2GM}{R} - kc^2$$

Re-introducing scale factor as in $R = a R_0$

$$\Rightarrow \dot{R} = \dot{a} R_0$$

we have, $$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2}}$$

Friedmann Equation.

—— this is the "equation of motion" for the expansion of the universe. If we can solve this equation for $a(t)$, we then know how the size of the universe depend on time.

(We introduced this as the "big question" in lecture 2.)

Looking at the Friedmann equation, we need to answer two questions before we can attempt to solve it: 01-26-2016

(1) How to fix k?

(2) What is the cosmic density ρ?

Answer to (1) : Recent observations have shown that we live in a flat universe, i.e. $k=0$.

It corresponds to a universe with zero total energy: $E_{tot}=0$

OR $E_{kin} = -E_{pot}$.

Answer to (2): We can try to construct the total cosmic density with the components that we know.

$$\rho = \rho_{tot} = \underbrace{\rho_m}_{\text{matter}} + \underbrace{\rho_\Lambda}_{\text{dark energy}} + \underbrace{\rho_{rad}}_{\text{radiation}} + \cdots$$

$$\rho_m = \underbrace{\rho_B}_{\text{Baryonic}} + \underbrace{\rho_{DM}}_{\text{Dark Matter.}}$$

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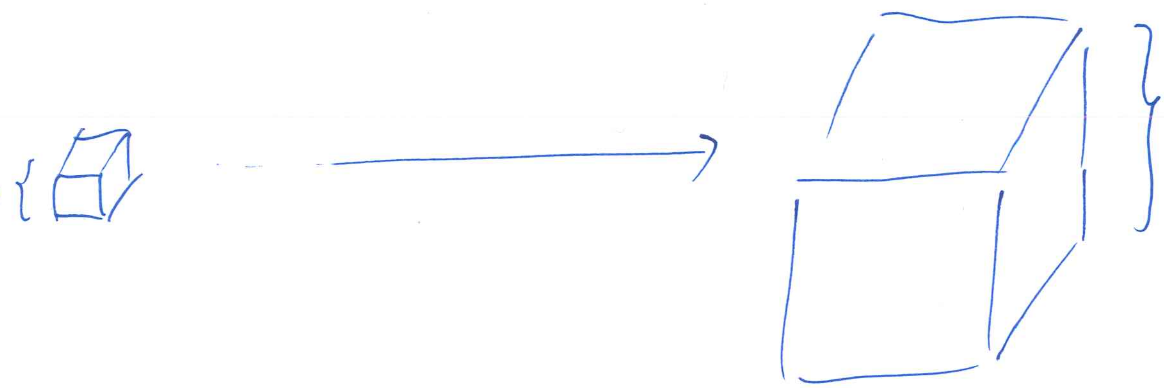
- Evolution of mass density.
  → conservation of mass.

aR_0  R_0

time = t time = t_0

The amount of mass ΔM in the box should be the same.

$$\therefore \quad \Delta M = \rho(t)\cdot(aR_0)^3 = \rho_0 \cdot R_0^3$$

$$\Rightarrow \quad \rho(t) = \rho_0 \cdot \left(\frac{1}{a(t)}\right)^3$$

$$= \rho_0 (1+z)^3 \quad [\text{using } 1+z = \frac{1}{a}]$$

For dark energy ('cosmological constant').

$$\rho_\Lambda(t) = \rho_{\Lambda,0} = \text{constant}.$$

For $t \gtrsim 10^4$ yr, we can neglect radiation. 01-26-2016.

$$\rho_{rad} \ll \rho_m$$

$$\therefore \rho_{tot} = \rho_m + \rho_\Lambda$$

We will end this lecture by introducing the density parameter Ω:

$$\left.\begin{aligned} \rho_{m,0} &= \Omega_m \cdot \rho_{tot,0} \\ \rho_{\Lambda,0} &= \Omega_\Lambda \cdot \rho_{tot,0} \end{aligned}\right\}$$

Recent measurements:

$\Omega_m = 0.3$

$\Omega_\Lambda = 0.7$

$\Omega_B = 0.05$