Last time we left off with the estimation of the luminosity of the first stars using the approximation

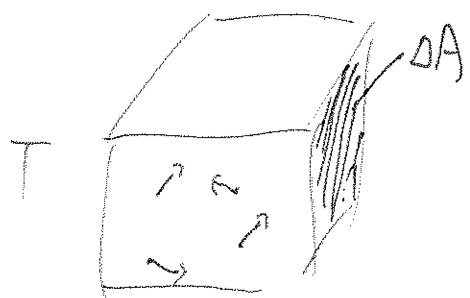
$$L \sim \frac{\Delta E}{\Delta t},$$

and argued that Δt can be estimated as the diffusion timescale for the photons.

This time we continue our quest in getting the estimates for L, and thus T_{eff}.

Structure of First Stars (continued).

→ Quick look at blackbody radiation



radiation energy density $u = \frac{\Delta E}{\Delta V} \propto T^4$

OR $u = a_{rad} \cdot T^4$

radiation flux through ΔA: $\text{flux} = \frac{\Delta E}{\Delta A \cdot \Delta t} = \sigma_{SB} \cdot T^4$

Also, $a_{rad} = \frac{4\sigma_{SB}}{c}$

$$4\pi R^2 \cdot \sigma_{SB} \cdot T^4 = L = \frac{\Delta E}{\Delta t}$$

"Stars are leaky photon boxes"

let's write down expressions for ΔE & Δt:

ΔE = total radiation energy inside star

$$= u \cdot V \sim u \cdot R^3$$

$$= a_{rad} \cdot T_I^4 \cdot R^3$$

[Recall $T_I \simeq 10^8 K$ for Pop III stars].

$\Delta t \hat{=}$ time needed for photon to leak out

$$= t_{RW} \sim t_{diff} \simeq \frac{N_{sc} \cdot \lambda_{mfp}}{c}$$

[Recall $R^2 = N_{sc} \cdot \lambda_{mfp}^2$

$$\Rightarrow \lambda_{mfp} = \frac{1}{n_e \cdot \sigma_T} \sim 1\,cm$$]

$$\Rightarrow 4\pi R^2 \cdot \sigma_{SB} \cdot T_{eff}^4 = L \sim a_{rad} \cdot T_I^4\, R \cdot c \cdot \lambda_{mfp}$$

$$\Rightarrow T_{eff} \simeq \left(\frac{\lambda_{mfp}}{R}\right)^{\frac{1}{4}} T_I$$

Put in the numbers:

$$T_{eff} \sim \left(\frac{1\,cm}{5.6\times10^{10}}\right)^{\frac{1}{4}} T_I$$

$$\sim \frac{1}{10^3} \cdot T_I \sim 10^5 K.$$

[Compare to $T_{eff,\odot} \sim 6000K$!]

$\Rightarrow$ Important for re-ionization of the Universe.

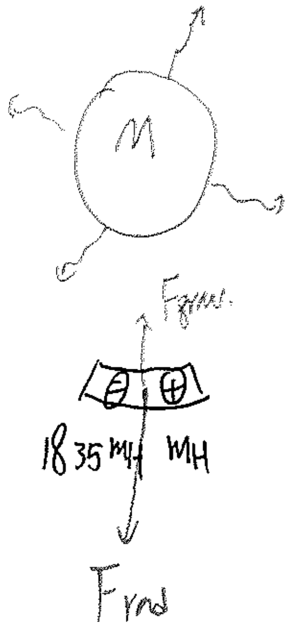
$$L = 4\pi R^2 \cdot \sigma_{SB} \cdot T_{eff}^4$$

$$\sim 10^6 L_\odot$$

Eddington luminosity

— Upper limit of luminosity that any star can have, without destroying itself.

— Photons carry momentum, if the luminosity is too high the radiation pressure would overcome gravity and blow up the star.



Require $F_{grav} \geq F_{rad}$

$$\Rightarrow \frac{G \cdot m_H M}{r^2} = F_{grav} \geq F_{rad} = \frac{\Delta p}{\Delta t}$$

$$= \frac{\Delta E}{c \cdot \Delta t}$$

$$= \frac{1}{c} \cdot \text{flux} \cdot \sigma_T$$

$$= \frac{1}{c} \cdot \frac{L}{4\pi r^2} \cdot \sigma_T$$

$$\Rightarrow \frac{G m_H M}{r^2} \geq \frac{1}{c} \cdot \frac{L}{4\pi r^2} \cdot \sigma_T$$

Pick the equality and label L as L_{Edd}; we have

$$\boxed{L_{Edd} = \frac{4\pi c G \cdot m_H \cdot M}{\sigma_T}}$$

— Eddington luminosity

$$L_{Edd} \simeq 10^{38} \text{ erg s}^{-1} \left(\frac{M}{M_\odot}\right)$$

For Pop III, $L_{Edd} \cong 10^{40}$ erg s^{-1}

Compare with the value we got,

$$L_{pop\,III} \cong 10^6 L_\odot \sim 4 \times 10^{39} \text{ erg s}^{-1}!$$

$$\therefore L_{pop\,III} \lesssim L_{Edd}.$$