AST353 Astrophysics. 04-14-2016

In the last lecture we introduced the equation of State, i.e. the ideal gas law

$$P = n k_B T = \rho \cdot \frac{k_B T}{m_H},$$

and argued that it can be re-written in the form of

$$P = P(\rho) = k \cdot \rho^{\frac{4}{3}}$$

for massive, Pop III stars. This allows us to close the system of 3 equations with 3 unknowns:

$$\begin{cases} \text{(i)} \quad \dfrac{dm}{dr} = 4\pi r^2 \cdot \rho(r) & \text{(mass conservation)} \\ \text{(ii)} \quad \dfrac{dP}{dr} = -g\rho(r) = -\rho \cdot \dfrac{G\,m(r)}{r^2} & \text{(Hydrostatic equilibrium)} \\ \text{(iii)} \quad P = K \cdot \rho^{\frac{4}{3}} & \text{(Equation of state)} \end{cases}$$

$\Rightarrow$ We can then in principle solve for the structure of the Pop III stars!

In this lecture, we would like to derive some estimates for the properties of first stars:

e.g. Luminosity L & surface temperature T_{eff}

Quiz 14 : recap. 04-14-2016

For stars we can almost always assume hydrostatic equilibrium

$$\frac{dP}{dr} = -g\rho = -\rho \frac{GM}{r^2}$$

$$P = P_{gas} + P_{rad} \simeq P_{rad} = \frac{1}{3} a_{rad} T^4$$
$$\simeq a_{rad} T^4.$$

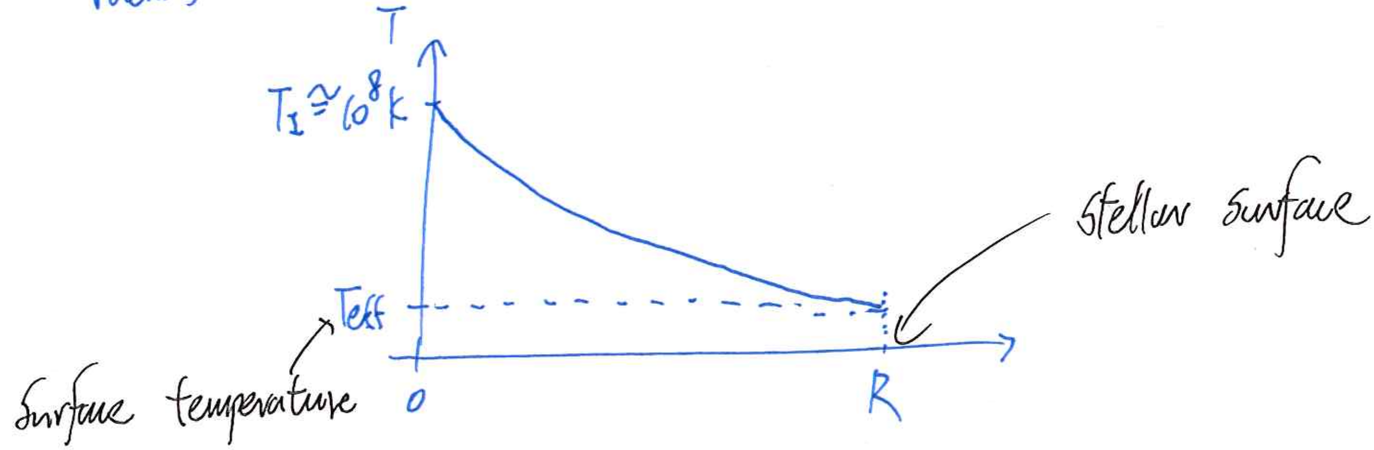
Say T_I: interior temperature

(Approximating $\frac{dP}{dr} \sim \frac{P}{R}$)

$$\frac{a_{rad} \cdot T^4}{R} \sim \frac{dP}{dr} \sim \frac{GM^2}{R^5}$$ (using hydrostatic eqm.)

$$\Rightarrow T_I \sim 10^8 K$$ (Compare with the Sun $T_{c,\odot} \sim 10^7 K$)

In reality, there is a dependence of temperature on radius



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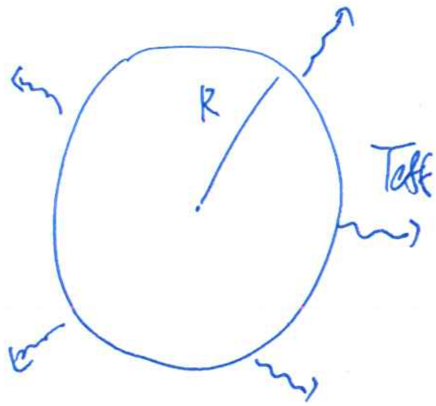

$$L = 4\pi R^2 \cdot \sigma_{SB} \cdot T_{eff}^4$$

(Stefan - Boltzmann law)

Q: How do we figure out the temperature gradient from the stellar interior to the surface?

Recall the three laws of stellar structure:

(i) Stars are almost always in Hydrostatic equilibrium!

$$\left[\frac{dP}{dr} = -g\rho \right]$$

(ii) Stars are well-regulated nuclear fusion reactors.

(iii) Stars are "leaky boxes" of photons.

Let's ponder on (iii) for a moment.

We can deduce the luminosity 04-14-2016

$$L = 4\pi R^2 \cdot \sigma_{SB} \cdot T_{eff}^4$$

by considering how photons, which are generated in the core of a star, make their ways to the surface.

i.e. $L \sim \frac{\Delta E}{\Delta t}$, where

ΔE = Total energy contained in photons in the star.

Δt = time to get the radiation out.

If we somehow get a handle on ΔE and Δt,

$\frac{\Delta E}{\Delta t} \sim L = 4\pi R^2 \cdot \sigma_{SB} \cdot T_{eff}^4$ will lead us to T_{eff}.

Turns out the energy density of radiation is closely related to the radiation pressure in massive stars:

$$\frac{\Delta E}{\Delta V} = a_{rad} \cdot T^4 \quad (\sim P_{rad})$$

Note: energy density and pressure have the same unit.

$\Delta V \sim R^3$ for stars, so we have ΔE.

→ Now, have to figure out

Δt.

Δt = time for photon to escape star via random walk (RW) 04-14-2016

$$\simeq t_{RW} = \underbrace{t_{diff}}_{\text{diffusion}} = \frac{N_{sc} \cdot \lambda_{mfp}}{c}$$ number of scattering

Recall from Lecture 16,

$$\lambda_{mfp}^2 \cdot N_{sc} = R^2 \quad \text{[RW equation]}.$$

$$\therefore \Delta t \sim \frac{N_{sc} \cdot \lambda_{mfp}}{c} = \frac{R^2}{\lambda_{mfp} \cdot c},$$

photon mean free path

and $\lambda_{mfp} = \lambda_\gamma = \dfrac{1}{n_e \cdot \sigma}$

n_e = free electron number density $= \dfrac{\rho}{m_H}$.

σ = cross-section for photon-electron interaction

$= \sigma_T$ (Thomson cross-section)

$= 0.66 \times 10^{-24}\ \text{cm}^2$.

Now we have Δt (assuming we have R and $\rho \sim \frac{M}{R^3}$ already) !!