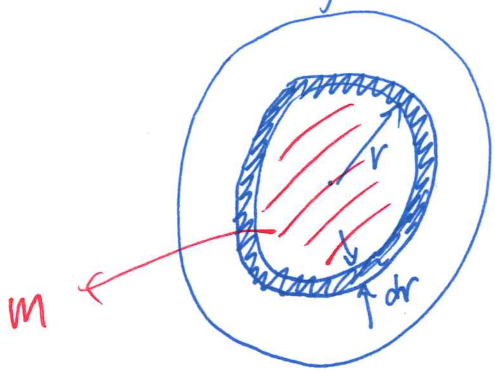
Last lecture, we introduced the concept of hydrostatic equilibrium. This lecture we continue our effort in deriving the structure of stars.

Structure of stars (Continued).



Basic equation so far:

(i) $$\frac{dm}{dr} = 4\pi r^2 \cdot \rho(r)$$

(ii) $$\frac{dP}{dr} = -g\rho = -\rho \cdot \frac{Gm}{r^2}$$

→ 2 equations.

Q: how many unknown?

$m(r)$, $P(r)$, $\rho(r)$ → 3.

We need another equation to close the system.

Equation of State (E.O.S.) 04-12-2016

→ ideal gas $P = n k_B T = \rho \cdot \frac{k_B T}{m_H}$.

→ We introduced another unknown T.
But there is one situation where one can solve the mechanical structure of star without knowing $T(r)$!

• Pressure inside stars.

$$\boxed{P = P_{gas} + P_{rad}} + \underbrace{P_{QM}}_{}$$

for first stars.

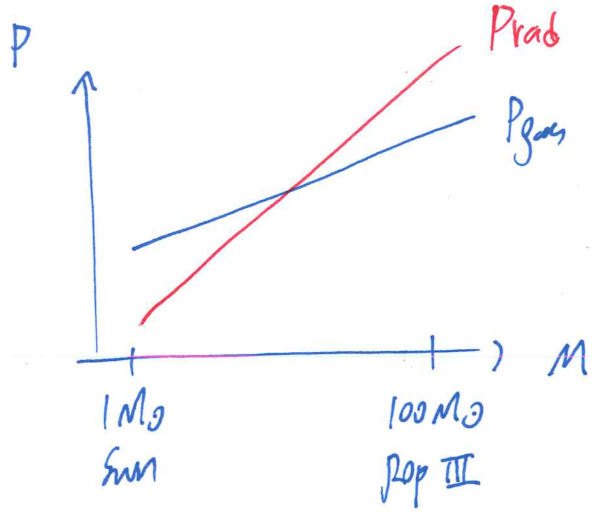
Degeneracy pressure. Important in white dwarfs, neutron stars.

$P_{gas} = n \cdot k_B T$

$P_{rad} = \frac{1}{3} a_{rad} \cdot T^4$, a_{rad} = radiation constant
$= 7.57 \times 10^{-15}$ erg $cm^{-3} \cdot K^{-4}$.

For blackbody radiation,

energy flux $= \frac{\Delta E}{\Delta A \cdot \Delta t} = \sigma_B T^4$ —— Stefan - Boltzmann law.



For Pop III $\qquad P = P_{gas} + P_{rad}$

$$P_{gas} = \beta P \qquad 0 < \beta < 1$$

$$P_{rad} = (1-\beta) P$$

$\beta \simeq 0.1$ for massive stars.

$$\rho \frac{k_B T}{m_H} = P_{gas} = \beta P = \left(\frac{\beta}{1-\beta}\right) P_{rad} = \left(\frac{\beta}{1-\beta}\right) \cdot \frac{1}{3} a_{rad} \cdot T^4$$

$$\rho \propto T^3 \quad \text{OR} \quad T \propto \rho^{\frac{1}{3}}$$

$$\Rightarrow \quad P = \rho \frac{k_B T}{m_H} \propto \rho^{\frac{4}{3}}$$

$$\text{i.e.,} \quad P = K \rho^{4/3}$$

→ crank out numerical solution:

$$R_{pop\,III} \sim 5 R_\odot$$

OR $\left[R_{pop\,III} \simeq 5 R_\odot \left(\frac{M}{100 M_\odot}\right)^{\frac{1}{2}} \right]$