THE FIRST STARS: Stellar Structure

- From cosmology + physics of SF, we predict: $M_{Pop\,III} \sim 100\,M_\odot$

$\Rightarrow$ Q's: $\left\{ \begin{array}{l} \text{radius } R = \\ \text{surface (effective) temperature } T_{eff} = \\ \text{Luminosity } \left(\frac{\text{energy}}{\text{time}}\right) \; L = \\ \text{lifetime } t_* = \end{array} \right.$?

→ Let's derive fundamental equations of stellar structure!

- Mechanical Structure → equation of motion (e.o.m.)?

e.o.m.: (Newton 2nd)

$$\boxed{\text{total force} = \text{mass} \times \text{acceleration}}$$

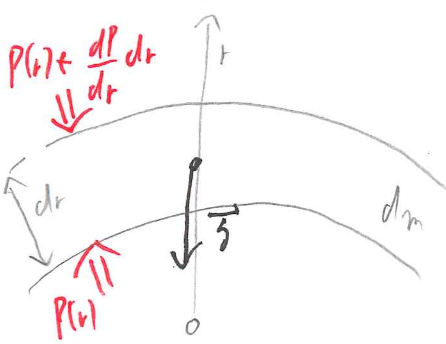
→ define mass coordinate:

$$dm = 4\pi r^2 \, dr \, \rho(r) \qquad (\text{mass of shell})$$

enclosed mass

$$m(r) = 4\pi \int_0^r r'^2 \rho(r') \, dr'$$

→ figure out total force:



$$\text{gravity} = -g(r)\, dm = -\frac{G\, m(r)}{r^2}\, dm$$

$$\text{pressure} = 4\pi r^2 P(r) - \left[P(r) + \frac{dP}{dr} dr \right] 4\pi r^2$$

$$\text{total force} = -g\, dm - \frac{1}{\rho}\frac{dP}{dr} \overbrace{4\pi r^2 \rho \, dr}^{dm} = \left[-g - \frac{1}{\rho}\frac{dP}{dr} \right] dm$$

$$\text{mass} \times \text{acceleration:} \quad dm \frac{d^2 r}{dt^2}$$

$\Rightarrow$ e.o.m.: $$\boxed{\frac{d^2 r}{dt^2} = -g - \frac{1}{\rho}\frac{dP}{dr}}$$

$\rightarrow$ "thought experiment": switch off pressure term

$$\frac{d^2 r}{dt^2} \simeq -\frac{R}{t_{ff}^2} \simeq -g \simeq -\frac{GM}{R^2}$$

$$\Rightarrow \quad t_{ff} \simeq \frac{1}{\sqrt{G\frac{M}{R^3}}} \simeq \frac{1}{\sqrt{G\rho}}$$ (free-fall timescale)

• NOTE: for Sun

$$\left.\langle\rho_\odot\rangle = \frac{M_\odot}{\frac{4\pi}{3}R_\odot^3} \sim 1\ g\ cm^{-3}\right\} \quad t_{ff} \simeq 1\ h\ !$$

$\rightarrow$ But: Sun has NOT changed for a long time!

$\Rightarrow$ pressure = gravity

• hydrostatic (mechanical) equilibrium $\left(\frac{d}{dt} = 0\right)$

$$\boxed{\frac{dP}{dr} = -\rho g}$$

- Simple estimate for central pressure, P_c, in stars:

at surface

$$\frac{dP}{dr} \simeq \frac{\cancel{P_0}^{\,0} - P_c}{R - 0} \simeq -\frac{P_c}{R}$$

$$-\rho g \simeq -\frac{M}{R^3}\frac{GM}{R^2} \simeq -\frac{GM^2}{R^5}$$

$$\Rightarrow P_c \simeq \frac{GM^2}{R^4}$$

e.g. Sun: $P_{c,\odot} \simeq 10^{16}$ dyn cm^{-2}

[cf. atmospheric pressure: $P_{atm} \simeq 10^5$ Pa $\simeq 10^6$ dyn cm^{-2}]