Physics of Star Formation (continued)

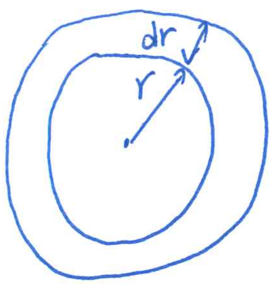
In this lecture, we'd like to complete the story of star formation by deriving the mass accretion rate in the disk $\dot{M}_{acc}$.

"fluid" (gas) viscosity → a.k.a. friction.

$$t_{friction} \sim t_{vis} \sim t_{RW} \sim \frac{R^2}{\lambda_{mfp} \cdot \bar{V}} \sim \frac{R^2}{\lambda_{mfp} \cdot C_s}$$

$$\Rightarrow t_{vis} \cong \frac{R^2}{\boxed{\lambda_{mfp} \cdot C_s}}$$ → This measures the strength of viscosity

'viscosity' $= \nu_{vis} = \lambda_{mfp} \cdot C_s$ [length²/time]



$$\Sigma = \frac{mass}{area}$$

$$\Delta m = \Sigma \cdot 2\pi r\, dr$$

$$\dot{M}_{acc} = 2\pi r \cdot \frac{dr}{dt} \cdot \Sigma$$

$$= 2\pi r \cdot V_r \cdot \Sigma$$

Knowing $\dot{M}_{acc} = 2\pi r \cdot V_r \cdot \Sigma$, 03-24-2016

we just need to find out what V_r is in order to calculate $\dot{M}_{acc}$.

- Torque $= \frac{dL}{dt}$ L: angular momentum.

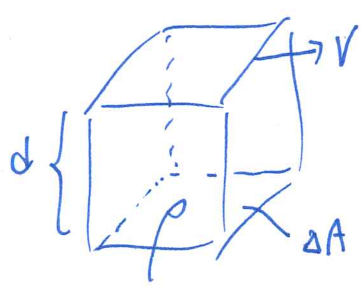
$$L \simeq r \cdot \Delta m \cdot V_{rot}$$

$$= 2\pi r^2 \cdot \Delta r \, \Sigma \cdot V_{rot}$$

$$\Rightarrow \frac{dL}{dt} = \Delta m \cdot \frac{dr}{dt} \cdot V_{rot}$$

$$\propto \Delta m \cdot V_{rot} \cdot V_r .$$

Also, $\tau = r \cdot F_{vis}$ F_{vis}: viscous force.



upper plate is moving at velocity V.

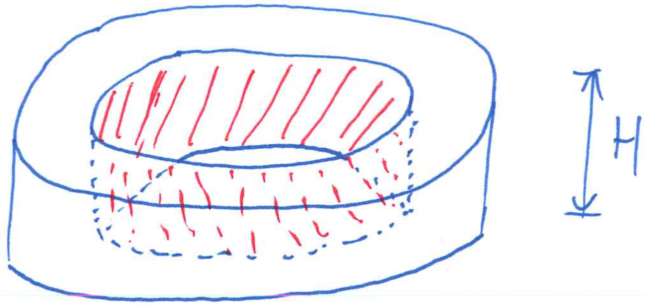
$$F_{vis} \propto \rho \cdot \frac{V}{d} \cdot \Delta A$$

$$\Rightarrow F_{vis} = \nu_{vis} \, \rho \cdot \frac{V}{d} \cdot \Delta A$$

→ Apply to protostellar accretion disk.

$$V = V_{rot}$$

$$d = \Delta r \sim r$$



$$\Delta A = 2\pi r H$$

Plug into $\quad r \cdot F_{vis} = \frac{dL}{dt} = \Delta m \cdot V_{rot} \cdot V_r$

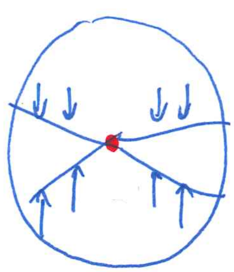
$$\boxed{\nu_{vis} = r V_r}$$

$$\Rightarrow \dot{M}_{acc} = 2\pi \Sigma \cdot r \cdot V_r$$
$$= 2\pi \nu_{vis} \cdot \Sigma$$

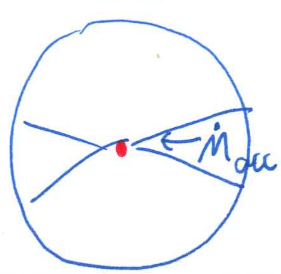
Star formation → Schematically.

① 

- cloud with mass $M > M_J$ (Jeans mass)
  - → collapses due to gravitational instability

② 

- initial angular momentum (spin) causes the collapsing cloud to form a disk.
- Because of rotation, the gas cannot fall directly to the star.

③ 

- viscosity (friction) induces an inward radial velocity of the gas in the disk
- grow protostar by $\dot{M}_{acc} = 2\pi \nu_{vis} \cdot \Sigma$

→ Let's calculate how quick viscosity is in our disk. 03-24-2016

$$t_{vis} = R^2 / \lambda_{mfp} \cdot C_s$$

$R = 100\ AU$; $AU = 1.5 \times 10^{13}\ cm$

$C_s \simeq \left(\frac{k_B \cdot T}{m_H} \right)^{\frac{1}{2}} = 10^4\ cm\,s^{-1}$

$T = 100k$

$\lambda_{mfp} = \frac{1}{n \cdot \sigma}$

From observation: $n \sim 10^8\ cm^{-3}$

$\sigma \simeq \pi r_0^2$

where $r_0 \triangleq$ radius of H atom

$= 1 \mathring{A}$

$= 10^{-8}\ cm.$

$$\therefore t_{vis} = \frac{10^{26} \cdot 10^4}{10^8 \cdot 10^4}$$

$$= 10^{18} s \sim 10^{11} yr\ !!$$

Observations show that the life time of protostellar disk $t_{disk} \sim 10\ Myr.$

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⇒ Big Problem $t_{vis} \gg t_{disk}$

→ Need source of "anomalous viscosity".

Some ideas:

(1) ramp up ν_{vis}

(2) Turbulent viscosity

(3) Magnetic fields

→ magnetic braking
