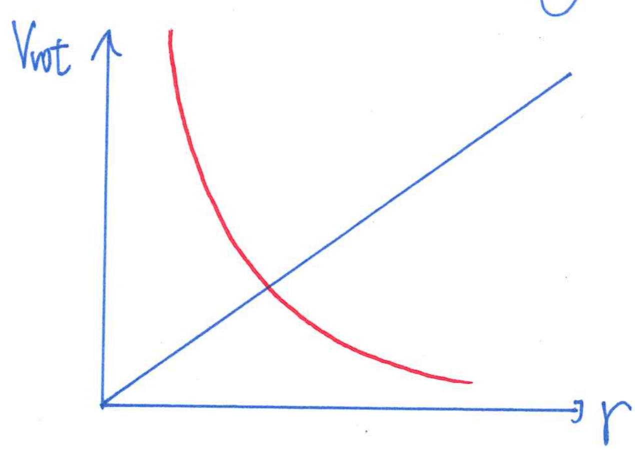
AST 353 Astrophysics. 03-22-2016

We pick up from last lecture and continue the task of figuring out the radial velocity V_r in the protostellar disk caused by friction.). First we'd like to know the frictional timescale t_{vis}.

- differential rotation of disk

Q: why does the disk NOT rotate (like a CD) as a solid body?

How is the disk rotating?



—— : solid body, Ω = const.

$V_{rot} = \underbrace{\Omega}_{\text{angular velocity}} \cdot r$

—— : Keplerian rotation.

Note: In the Keplerian case,

$\frac{d\Omega}{dr} \neq$ constant $\Rightarrow$ frictional force.

$\rightarrow$ inward motion

$\Rightarrow$ growth of protostars!

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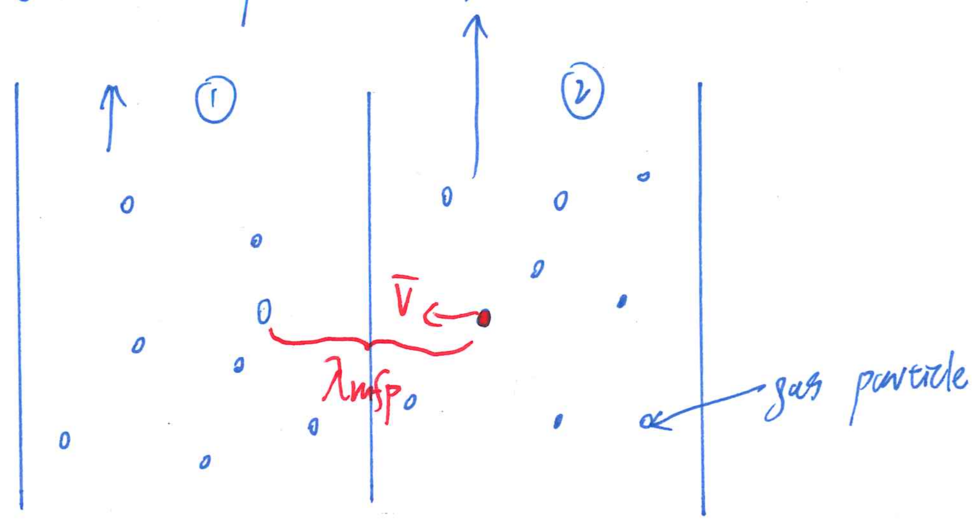
- Friction in fluids (gas in astronomy)
  → "viscosity"

Now we wish to figure out the timescale at which friction in the disk works. To do this, we consider an idealized cartoon picture of a shear flow:



The two layers of gas (1 and 2) are moving adjacent to each other with <u>different</u> velocities.

The viscosity of the gas is due to the excursion of gas particles from one layer to another.

$\lambda_{mfp} \cong$ mean free path
$\equiv$ average distance a gas particle travel between collisions.

$$\bar{v} \sim C_s = \sqrt{\frac{k_B T}{m_H}} = \text{speed of sound.}$$

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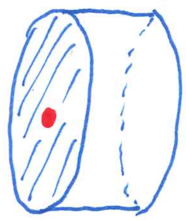
• Mean-free path.



area of circle of influence $\cong \sigma$ = cross section.

$$\Delta V = \sigma \cdot \lambda_{mfp}$$

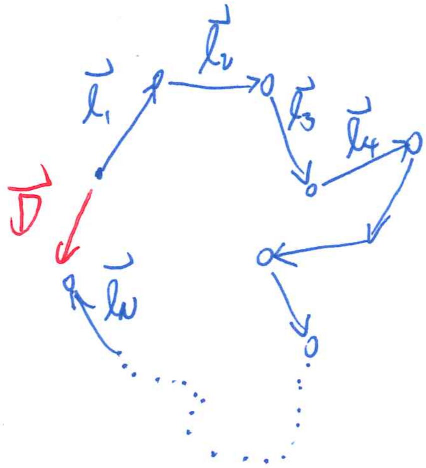
n: number density

$$\Delta V \cdot n = 1 \quad \text{(by definition of mfp)}$$

$$\Rightarrow \quad \lambda_{mfp} = \frac{1}{\sigma \cdot n}$$

• Viscous timescale

→ random walk (RW)



assume:

$$|\vec{l}_1| = |\vec{l}_2| = |\vec{l}_3| = \cdots = |\vec{l}_N|$$

Net displacement $\vec{D} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_N$

$$\langle \vec{D} \rangle = \langle \vec{l} \rangle = 0$$

($\because$ isotropic scattering)

$$\vec{D}^2 = (\vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \cdots + \vec{l}_N)^2$$

$$\langle D^2 \rangle = \langle \vec{l}_1^{\,2} \rangle + \langle \vec{l}_2^{\,2} \rangle + \cdots + \langle l_N^2 \rangle + \underbrace{\langle \vec{l}_1 \cdot \vec{l}_2 \rangle + \cdots}_{=0}$$

$$\Rightarrow \quad \boxed{D^2 = N \cdot \lambda_{mfp}^2} \quad \text{OR} \quad \boxed{D = \sqrt{N} \cdot \lambda_{mfp}}$$

So, to travel a distance L, we need 03-22-2016

$$L = \sqrt{N} \cdot \lambda_{mfp}$$

$$t_{RW} = t_{friction} = t_{vis}$$

$$= \frac{N \cdot \lambda_{mfp}}{c_s}$$

$$= \left(\frac{L^2}{\lambda_{mfp}^2}\right) \cdot \frac{\lambda_{mfp}}{c_s}$$

$$\boxed{t_{vis} = L^2 / \lambda_{mfp} \cdot c_s}$$

$L = R$ for a protostellar disk