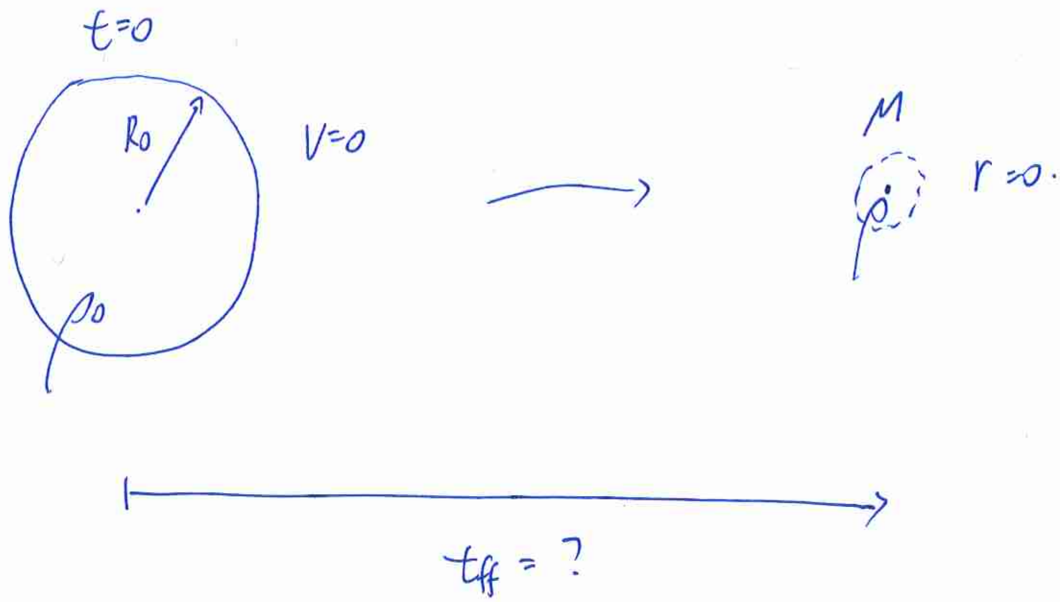
Star Formation, Quick Intro. (continued). 03-10-2016

- Free-fall collapse.



We want to figure out the "free-fall time (scale)".

$$M = \frac{4}{3}\pi R_0^3 \cdot \rho_0 = \frac{4}{3}\pi r^3 \cdot \rho = \text{constant}.$$

E.o.m.
$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2} + \cancel{\left(\text{pressure force}\right)}$$

Ignored for free-fall collapse.

(i) Precise calculation

$$\frac{d}{dt}\left[\left(\frac{dr}{dt}\right)^2\right] = 2\cdot\frac{dr}{dt}\cdot\frac{d^2r}{dt^2}$$

$$= -\frac{2GM}{r^2}\cdot\left(\frac{dr}{dt}\right)$$

$$\Rightarrow \int_0^{V=V(r)} d\left[\left(\frac{dr}{dt}\right)^2\right] = -2GM\int_{R_0}^{r}\frac{dr}{r^2}$$

$$\left(\frac{dr}{dt}\right)^2 = 2GM\left[\frac{1}{r} - \frac{1}{R_0}\right]$$

infall

$$\Rightarrow \quad \frac{dr}{dt} = (-)\sqrt{2GM}\left[\frac{1}{r} - \frac{1}{R_0}\right]^{\frac{1}{2}}$$

→ Second integral

→ separation of variables.

$$t_{ff} = \int_0^{t_{ff}} dt = -\int_{R_0}^{0} \frac{dr}{\sqrt{2GM}\left[\frac{1}{r} - \frac{1}{R_0}\right]^{\frac{1}{2}}}$$

Substitute $x = r/R_0$,

$$\Rightarrow \quad t_{ff} = R_0^{3/2}/\sqrt{2GM} \cdot \underbrace{\int_0^1 \frac{\sqrt{x}\,dx}{\sqrt{1-x}}}_{\pi/2}$$

$$\boxed{t_{ff} = \frac{R_0^{3/2}}{\sqrt{2GM}} \cdot \frac{\pi}{2} = \sqrt{\frac{3\pi}{32 G \rho_0}}}$$

(iii) Shortcut via Order-of-magnitude estimate. 03-10-2016

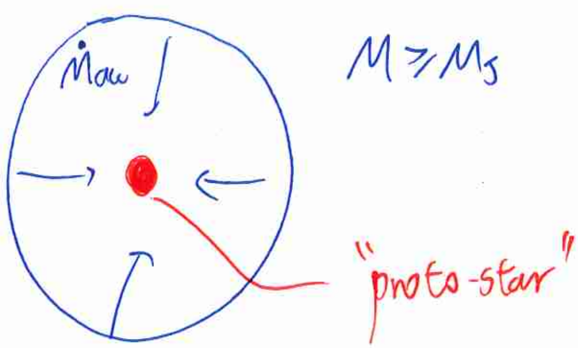
$$-\frac{R_0}{t_{ff}^2} \sim \frac{d^2r}{dt^2} = -\frac{GM}{r^2} \simeq \frac{GM}{R_0^2}$$

$$t_{ff} \simeq \sqrt{\frac{1}{GM/R_0^3}} \cong \sqrt{\frac{1}{G\rho_0}}$$

Note: shortcut got us within $\sqrt{\frac{32}{3\pi}} \simeq 1.84 < 2$ of precise results!

- Protostellar Growth.

(i) First, ignore angular momentum ('spin')



$M \gtrsim M_J$

$\dot{M}_{acc} \cong$ accretion rate.

$$\dot{M}_{acc} = M_J / t_{ff} = c_s^3 / G \propto T^{3/2}$$

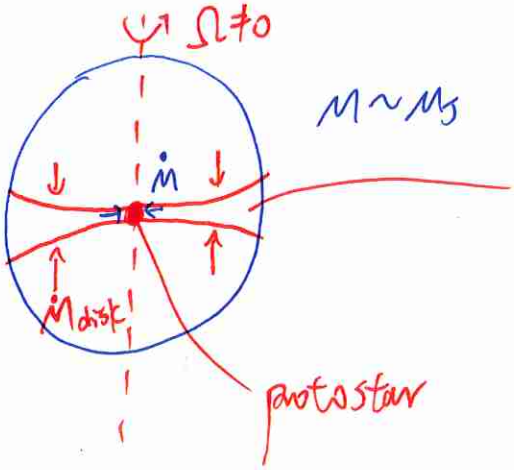
For MW (e.g. pre-solar nebula) 03-10-2016

$T = 10K$

$\Rightarrow \dot{M}_{acc} \cong 10^{-5} M_\odot yr^{-1}$

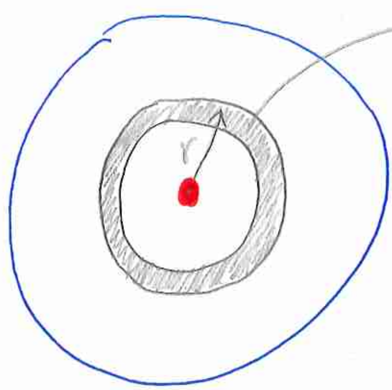
$\Rightarrow$ takes a few $10^5 yr$ to assemble the sun!

(ii) Now, we do include spin.



$\dot{M}_{disk} \simeq M_J / t_{ff}$ as before.

Bird's eye view of the disk:



$dM = \Sigma \cdot 2\pi r \cdot dr$

surface mass density

$\Sigma = M_{disk} / \pi R_{disk}^2$

(unit = $g\ cm^{-2}$)

$$dM = \Sigma \cdot 2\pi r \, dr$$

divide both sides by dt,

$$\dot{M} = \frac{dM}{dt} = 2\pi r \cdot \frac{dr}{dt} \cdot \Sigma$$

$$= 2\pi r \, V_r \, \Sigma$$

inward radial velocity of the gas in the disk moving toward the protostar.

It is caused by the 'gas friction' in the disk.

We will pick up from here next time.