AST 353 Astrophysics. 02-25-2016

Last time we figured out the properties of Virialized DM halos. The most important message from last lecture was:

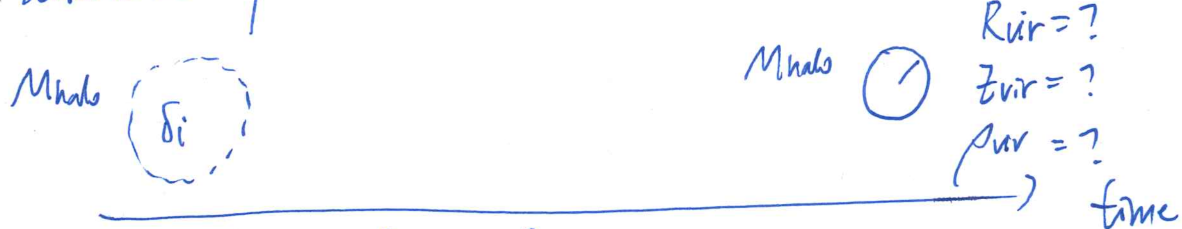
$$\rho_{vir} \cong 200\, \bar{\rho}(z = z_{vir})$$

In other words, when virialization happens, the density of the DM halo is about 200 times higher than its background.

— This time, we will practise the same ideas with an example.

Halo Virialization: Worked Example.

Schematic picture:



$\begin{cases} z_i = 1000, \quad \delta_i = 2\times10^{-2} \\ M_{halo} = 10^8 M_\odot \end{cases}$

Initial Conditions

Virialization

Our task is to figure out three numbers: 02-25-2016

R_{vir} , z_{vir} , and ρ_{vir}.

It turns out they are not independent,

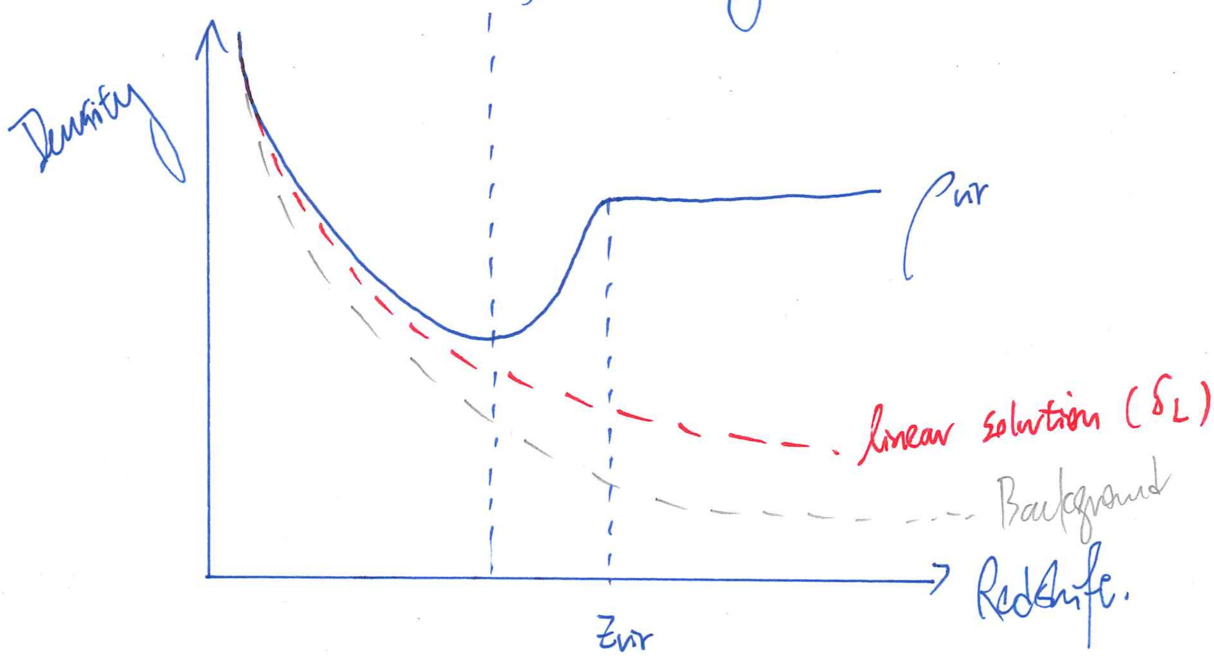
$$M_{halo} = \frac{4}{3}\pi R_{vir}^3 \cdot \rho_{vir}$$

$$\rho_{vir} \simeq 200 \cdot \bar{\rho}(z = z_{vir})$$

$$[\text{also } \bar{\rho}(z) = 3\times10^{-30}\,\mathrm{g\,cm^{-3}}\,(1+z)^3\,]$$

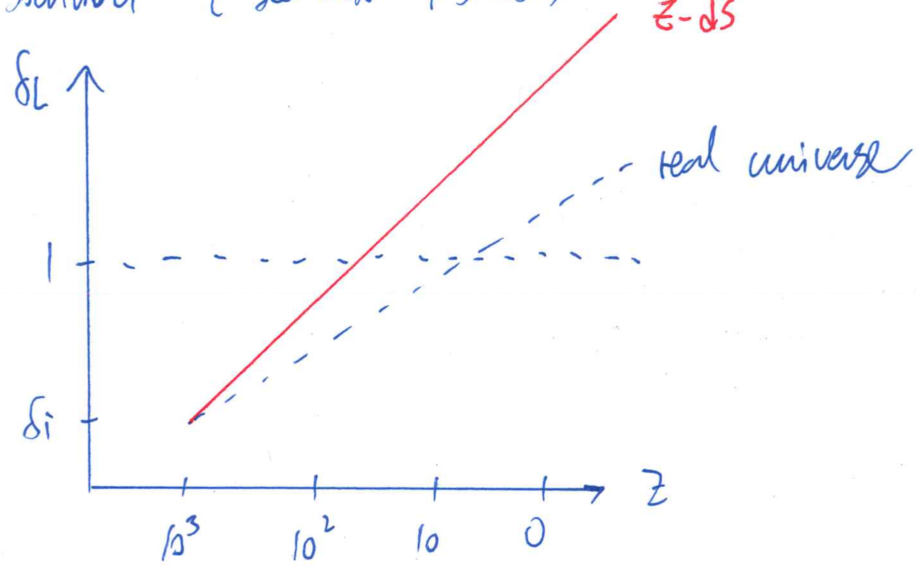
So our only unknown is z_{vir}.

Recall the time evolution of density:



- linear solution δ_L
  - → at turnaround $\delta_L \simeq 1$
  - at virialization $\delta_L = \delta_{crit} = 1.68 \simeq 2$.

- linear solution (see also PS #3)



Use E-dS solution

$$2 \simeq \delta_{crit} = \delta_{vir} = \delta_i \cdot \frac{1+z_i}{1+z_{vir}}$$

$$1+z_i = \frac{\delta_i}{2}(1+z_i)$$

$$= \frac{2\times10^{-2}}{2}(1+1000)$$

$$\simeq 10$$

$$\Rightarrow \quad z_{vir} \sim 9.$$

$$\therefore \rho_{vir} = 200\bar{\rho}(z_{vir})$$

$$= 6\times10^{-24}\ g\,cm^{-3}.$$

Putting in the numbers..

$$R_{vir} = \left(\frac{M_{halo}}{\frac{4}{3}\pi\rho_{vir}}\right)^{\frac{1}{3}} = 2\times10^{21}\ cm$$

$$\lesssim 1\ kpc.$$