Neutron Stars cont'd March 6, 2008

- **Structure of cold, dead stars**

  - construct e.o.s. for matter in the 'ground state'

    - zero energy ($\rightarrow$ zero T)
    - ALL nuclear fuel spent entirely

    $\Rightarrow$ cold, dead matter (cold 'catalyzed' matter)

$\rightarrow$ construct equilibrium models

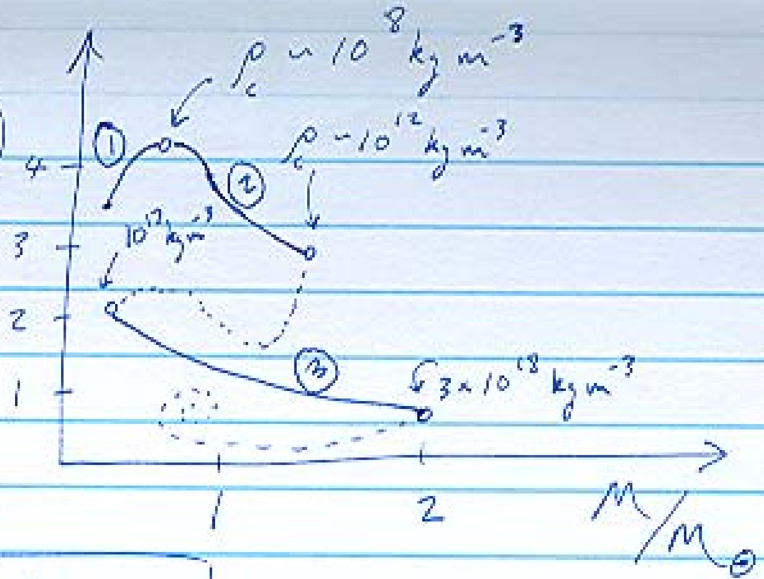
- choose central density, ρ_c
- calculate central pressure, P_c (from e.o.s.)

- integrate OV equation $\left(\frac{dP}{dr} = \cdots\right)$ from center $(r=0)$ to surface $(P=0)$ $(r=R)$

$\Rightarrow$ Find: $M = m(R) = M(\rho_c)$

$R = R(\rho_c)$, similarly

$\rightarrow$ plot mass-radius relation



configuration is
——— stable
- - - - - - - unstable

(1) Brown dwarfs and planets
(2) WD
(3) NS

→ stability requires $\frac{dM}{d\rho_c} > 0$

Q: Why is that?

Consider the unstable case $\left(\frac{dM}{d\rho_c} < 0\right)$

- <u>Simple estimate for R of cold, dead stars</u>

- consider WDs + NSs:

	Pressure due to	Mass	Gravity (E_{pot})	pressure (E_{kin})
WD	deg. elec. $n = n_e$	$\rho = 2m_H n_e \sim m_H n$	$\frac{GMm_H}{R}$	$m_e c^2$
NS	deg. neutrons $n = n_n$	$\rho = m_H n_n$	$\frac{GMm_H}{R}$	$m_H c^2$

$\}\ m_0 c^2$

→ assume degeneracy pressure at NR / UR boundary

$$\Rightarrow \quad \epsilon_{kin} = m_o c^2$$

→ For equilibrium, need $\epsilon_{pot} \simeq \epsilon_{kin}$

→ $M \simeq \rho R^3 \simeq m_H n R^3$ (same for WD & NS!)

→ fact that we have degenerate gas close to NR/UR boundary allows us to estimate n

$$\ell \simeq \lambda_{deB}$$

$$\begin{pmatrix}\text{mean particle} \\ \text{separation}\end{pmatrix} \simeq \begin{pmatrix}\text{de Broglie} \\ \text{wavelength}\end{pmatrix}$$

Compton wavelength

$$\lambda_{deB} = \frac{h}{p} \simeq \frac{h}{m_o c} \simeq \lambda_c$$

for NR / UR boundary, $p \simeq m_o c$

$$\Rightarrow \quad n^{-1/3} \simeq \frac{h}{m_o c}, \quad m_o = \begin{cases} m_e & \text{for WD} \\ m_H & \text{for NS} \end{cases}$$

$$\Rightarrow \quad M \simeq m_H \left(\frac{h}{m_o c}\right)^{-3} R^3$$

→ From $\epsilon_{pot} \simeq \epsilon_{kin}$

$$\Rightarrow \quad \frac{G m_H^2}{\hbar c} \left(\frac{h}{m_o c}\right)^{-3} R^2 \simeq \frac{m_o c^2}{\hbar c}$$

Now, $\alpha_G = \frac{G m_H^2}{\hbar c}$, from before

gravitational fine-structure constant $(\alpha_G \sim 10^{-38})$

Solve for radius:

$$\boxed{R = \alpha_G^{-1/2} \lambda_c = \alpha_G^{-1/2} \frac{h}{m_0 c}}$$

compare with $M_{ch} = \alpha_G^{-3/2} m_H$

→ For WD: $m_0 = m_e$

$\Longrightarrow R \simeq 10{,}000$ km

→ For NS: $m_0 = m_H$

$\Longrightarrow R \simeq 10$ km

→ Note $\frac{R_{WD}}{R_{NS}} \simeq \frac{m_H}{m_e} \equiv 1{,}836$

→ microphysics determines macrophysics!