Neutron Stars (Cont'd) March 4, 2008

• Neutron degeneracy pressure

- n's become degenerate (NR) at $\rho \sim 10^{16}\ kg\,m^{-3}$

$\Rightarrow$ (NR) degenerate gas pressure (degeneracy pressure)

$$P = \frac{h^2}{5m_0}\left(\frac{3}{8\pi}\right)^{2/3} n^{5/3}$$

- Here $m_0 = m_n \simeq m_H = 1.67 \times 10^{-27}\ kg$

$n = n_n \hat{=}$ neutron number density

$$\Rightarrow n_n = \frac{\rho}{m_H}$$

- n's become UR degenerate

$$m_H c^2 \simeq p_F c = h n_n^{1/3} c$$

$$\Rightarrow \rho = 5 \times 10^{17}\ kg\ m^{-3}$$

• Upper Mass Limit

- recall: for a NS,

$$\frac{R_s}{R} = \frac{2GM}{c^2 R} \sim 0.6$$

$\Rightarrow$ GR is crucial!

$\rightarrow$ Describe mechanical structure of NS with OV equation:

$$\frac{dP}{dr} = -\rho \frac{GM}{r^2}\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi r^3 P}{mc^2}\right)\left(1 - \frac{2GM}{c^2 r}\right)^{-1}$$

$\rightarrow$ Consider idealized case ($\rho = \rho_0 =$ constant)
"incompressible case"

$$\Rightarrow m(r) = \frac{4\pi}{3} r^3 \rho_0 \;;\; M = \frac{4\pi}{3} R^3 \rho_0 = m(R)$$

- Separation of variables

$$\int_{P(r)}^{0} \frac{dP}{\left(1 + \frac{P}{\rho_0 c^2}\right)\left(1 + \frac{3P}{\rho_0 c^2}\right)} = -\frac{4\pi G \rho_0^2}{3} \int_{r}^{R} \frac{r\,dr}{1 - \frac{8\pi G \rho_0}{3c^2} r^2}$$

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$$\Rightarrow -\frac{1}{2}\rho_0 c^2 \left[\ln \frac{1+\frac{P}{\rho_0 c^2}}{1+\frac{3P}{\rho_0 c^2}} \right]\Bigg|_{P(r)}^{0}$$

$$= \frac{1}{4}\rho_0 c^2 \left[\ln\left(1 - \frac{8\pi G \rho_0}{3c^2} r^2\right) \right]\Bigg|_{r}^{R}$$

$$\Rightarrow \frac{1+\frac{P}{\rho_0 c^2}}{1+\frac{3P}{\rho_0 c^2}} = \left(\frac{1 - \frac{2GM}{c^2 R}}{1 - \frac{2GM}{c^2 R}\frac{r^2}{R^2}} \right)^{1/2}$$

$\rightarrow$ recall $R_s \equiv \dfrac{2GM}{c^2}$

$$\Rightarrow \boxed{P(r) = \rho_0 c^2 \frac{\sqrt{1 - \frac{R_s r^2}{R^3}} - \sqrt{1 - \frac{R_s}{R}}}{3\sqrt{1 - \frac{R_s}{R}} - \sqrt{1 - \frac{R_s r^2}{R^3}}}}$$

→ Evaluate central pressure

$$P_c = P(r=0)$$

$$P_c = \rho_0 c^2 \, \frac{1 - \sqrt{1 - \frac{R_s}{R}}}{3\sqrt{1 - \frac{R_s}{R}} - 1}$$

Compare to Newtonian case → $P_c = \dfrac{2\pi G \rho_0^2}{3} R^2$

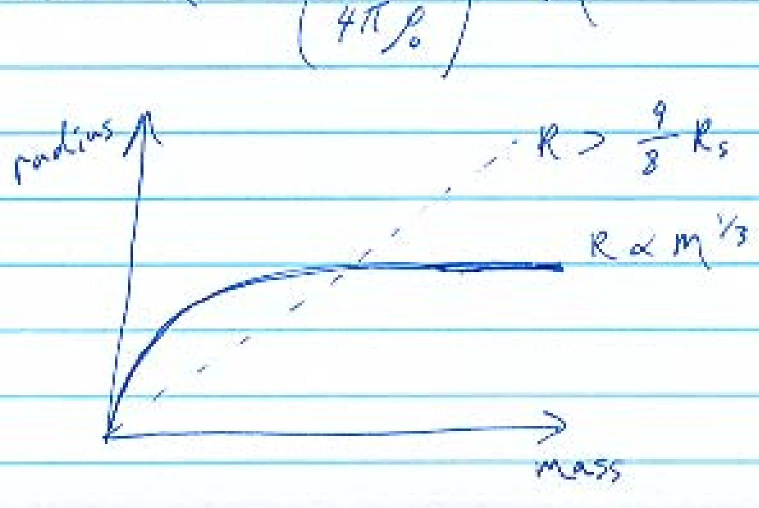
Notice : $R \to \frac{9}{8} R_s \Rightarrow P_c \to \infty$

Q: What does this imply?

A: $R > \frac{9}{8} R_s$ (infinite pressure does not exist!)

$\Rightarrow$ (i) $R > \frac{9}{8} R_s = \dfrac{9G}{4c^2} M$

(ii) $R = \left(\dfrac{3}{4\pi\rho_0}\right)^{1/3} M^{1/3}$



- Solve for M_{max} : $\frac{9}{4}\frac{G}{c^2}M_{max} = \left(\frac{3}{4\pi\rho_0}\right)^{1/3} M_{max}^{1/3}$

$$\Rightarrow \boxed{M_{max} = \frac{8}{27}\left(\frac{c^2}{G}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}}$$

Oppenheimer-Volkoff limit for mass of NS

For $\rho_0 \simeq 5 \times 10^{17}$ kg m^{-3}

$\Rightarrow M_{max} \simeq 5\, M_\odot$ (M_{OV})

Compare to Chandrasekhar mass limit for WDs - $M_{ch} = 1.4\, M_\odot$

• Upper mass limit (revisited)

→ Use "extreme equation of state" (EOS)
$\rho = \rho_0$ = state

EOS → $P = K\rho^\gamma$; K, γ are constants

$\Rightarrow \rho = \left(\frac{P}{K}\right)^{1/\gamma}$ ⟶ Have ρ = constant if $\gamma \to \infty$

(resistance to being compressed)

→ However: real equations of state are "softer" (γ is small)

→ gas can be compressed!

→ Within large uncertainties

$M_{max} = 1.5 - 3 M_\odot$
(M_{OV})

Note: All currently observed NS (from pulsars) have masses close to $1.45 M_\odot$ ($\gtrsim M_{ch} = 1.4 M_\odot$)