# Black Holes

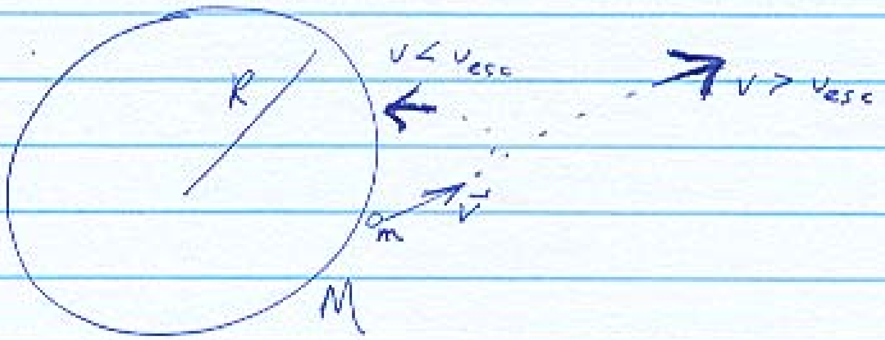
March 25, 2008

- **Early speculations**

→ late $18^{th}$ century (Enlightenment)
- John Mitchell (1783)
- Pierre Simon Laplace (Le Systéme du Monde) (1796)

→ **2 ingredients**

(1) Newtonian gravity
(2) corpuscular theory of light (also Newton's)




- Consider escape speed:

$$E_{tot} = \frac{-GMm}{r} + \frac{1}{2}mv^2$$

$$E_{tot} = 0 \implies v_{esc} = \sqrt{\frac{2GM}{r}}$$

→ for light particles $v_{esc} = c$

$\Rightarrow$ Critical radius $\boxed{R_s = \frac{2GM}{c^2}}$ (today called the Schwarzschild radius)

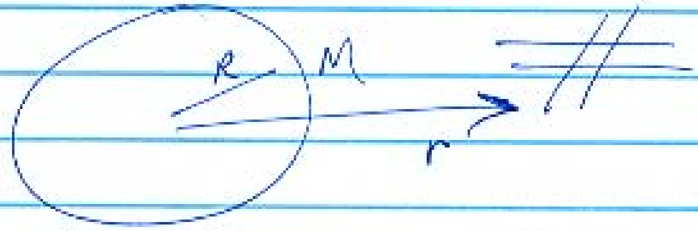
$\rightarrow$ Crucial difference from modern concept of black holes (BH)

- dark star would still be visible not-too-far-away from its surface

• <u>Schwarzschild geometry</u>

- Use GR to determine geometry (gravitational field) around a spherically symmetric object of mass M

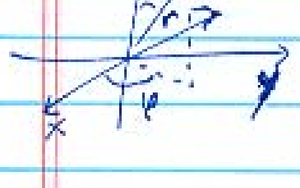
$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ & g_{11} & g_{12} & g_{13} \\ & & g_{22} & g_{23} \\ & & & g_{33} \end{pmatrix}$$




- <u>further</u>: assume no time dependence ("static")

- Choose spherical coordinates $(r, \theta, \varphi)$

$\Leftrightarrow$ spherical symmetry



$\Rightarrow$ spacetime metric must have general form

$$ds^2 = -A(r)\,c^2 dt^2 + B(r)\,dr^2 +$$
$$r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right)$$

$$\Longrightarrow \quad g_{\mu\nu} = \begin{pmatrix} -A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & r^2\sin^2\theta & 0 \\ 0 & 0 & 0 & r^2 \end{pmatrix}$$

Recall Einstein's field equation:

$$G_{\mu\nu}\left(\begin{matrix}\text{2nd derivatives} \\ \text{of } g_{\mu\nu}\end{matrix},\ \begin{matrix}\text{1st derivatives} \\ \text{of } g_{\mu\nu}\end{matrix},\ g_{\mu\nu}\right) = \frac{8\pi}{c^4} T_{\mu\nu}$$

For $T_{\mu\nu} = 0$, find vacuum solutions of GR!

$$A(r) = 1 - \frac{2GM}{c^2 r} = g_{00}$$

$$B(r) = \frac{1}{A(r)} = g_{11}$$

Schwarzschild Metric: (1916)

$$\Rightarrow \boxed{ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2 dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} + r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right)}$$

- interpretation of radial coordinate $r$

NOT measure of the physical distance
from center

→ 3D space is curved!

→ $r$ is a kind of 'circumferential radius'

→ Consider physical area of a surface at
$r = \text{const}$, $t = \text{const}$ :

$$A = \int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta \, d\theta \, d\varphi = 4\pi r^2$$

- interpretation of temporal coordinate $t$

→ Consider 2 stationary observers $(d\varphi = d\theta = dr = 0)$:

(A) at $r \rightarrow \infty$

(B) at $r$

→ figure out time that each observer measures:

(A) $r \rightarrow \infty$ , $d\tau_\infty = dt$

(B) $d\tau = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2} d\tau_\infty$

→ time slows down deep in gravitational field

("gravitational time dilation")

- effect on light:

(B) emits light with frequency $\nu_{em} = \frac{1}{d\tau}$

(A) receives photons with $\nu_{obs} = \frac{1}{d\tau_\infty}$

$$\boxed{\nu_\infty = \nu \left(1 - \frac{2GM}{c^2 r}\right)^{1/2}}.$$

("gravitational redshift")