Pulsars and Magnetars (cont'd) March 20, 2008

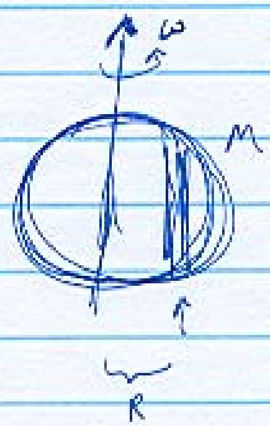
• Rotational Energy

$E_{rot} = \frac{1}{2} I \omega^2$, $\omega \hat{=}$ angular velocity

$= \frac{2\pi}{P}$ ← period

$I \hat{=}$ moment of inertia $= \int_0^R r^2 dm$

for solid sphere:



$I = \frac{2}{5} M R^2$

→ for typical NS parameters

$\begin{pmatrix} R = 10 \text{ km} \\ M = 1 M_\odot \end{pmatrix} \Rightarrow I = 10^{38} \text{ kg m}^2$

→ know from observations that pulsars slow down!

- e.g. Crab Pulsar: $\omega = \frac{2\pi}{P} = \frac{2\pi}{33 \text{ms}}$

$= 190 \text{ s}^{-1}$

- observed: $\frac{dP}{dt} = \frac{1\,ms}{90\,yr}$

$$\frac{d\omega}{dt} = \frac{-2\pi}{P^2}\frac{dP}{dt} = -2\times10^{-9}\,s^{-2}$$

$$\frac{dE_{rot}}{dt} = I\omega\frac{d\omega}{dt} = -4\times10^{31}\,W$$

Estimate pulsar age τ:

$$P \simeq 2\frac{dP}{dt}\tau \Rightarrow \boxed{\tau \equiv \frac{P}{2\dot{P}}}, \quad \dot{P} = \frac{dP}{dt}$$

("spin-down age")

- $\tau \sim 1{,}500$ yr, for the Crab pulsar.
  (true age ~ 950 yr)
- Compare with radiation output from Crab. $\longrightarrow$ $L_{Crab} = \frac{dE}{dt} = 5\times10^{31}$ W.

Big Q: How is rotation converted into radiation?

- Radiation mechanism

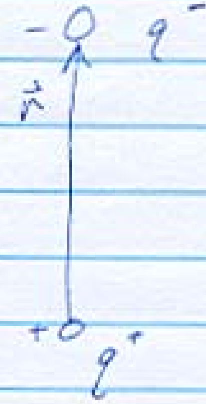
- in general: accelerated charges radiate

$$\Rightarrow \frac{dE}{dt} = \frac{1}{6\pi\epsilon_0 c^3} q^2 |\vec{a}|^2$$, Larmor's formula.

$\vec{a}$ — acceleration

$$\left(\epsilon_0 = 8.85 \times 10^{-12} \ C^2 N^{-1} m^{-2}\right)$$

– Consider an electric dipole

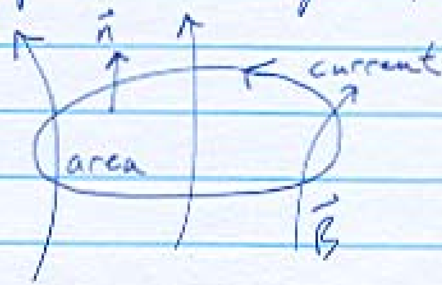


electric dipole moment $\vec{d} = q\vec{r}$

$$\boxed{\frac{dE}{dt} = \frac{1}{6\pi\epsilon_0 c^2} |\ddot{\vec{d}}|^2}$$

$\rightarrow$ electric dipole radiation

$\rightarrow$ magnetic analogue, "magnetic dipole moment"



$$\vec{\mu} = \text{area} \times \text{current } \hat{n}$$
$$= \pi R^2 \times i \quad \hat{n}$$

– from Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i \qquad \left(\mu_0 = 4\pi \times 10^{-7} \ NA^{-2}\right)$$

$\Rightarrow 2\pi R B = \mu_0 i$

$\Rightarrow \mu \equiv |\vec{\mu}| = \frac{1}{\mu_0} R^3 B$

$\rightarrow$ <u>magnetic dipole radiation</u>:

$$\boxed{\frac{dE}{dt} = \frac{1}{6\pi\epsilon_0 c^3} \frac{\mu_0}{4\pi} \left|\ddot{\mu}\right|^2}$$

- Consider sinusoidal variation

$$\mu = A \sin \omega t$$

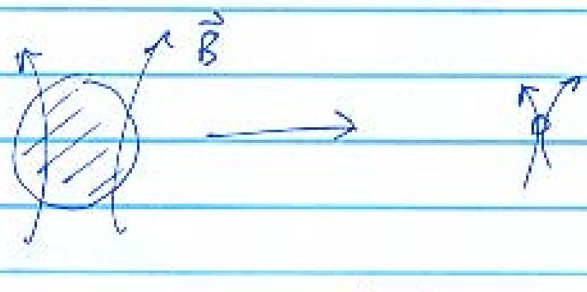
$$\ddot{\mu} = -\omega^2 \mu$$

$$\frac{dE}{dt} = \frac{2}{3c^3} \frac{\mu_0}{4\pi} \omega^4 \frac{R^6 B^2}{\mu_0^2}$$

For Crab, we find that $B = 10^9\,T$

Q: Where do such strong B-fields come from?

A: collapse of star!



- magnetic fields are "frozen" into fluid

$\rightarrow$ MHD (magnetohydrodynamics)

→ magnetic flux $= \phi = \text{area} \times B$
$= \pi r^2 B = \text{constant}$

$$B_f = B_i \left(\frac{r_i}{r_f} \right)^2$$

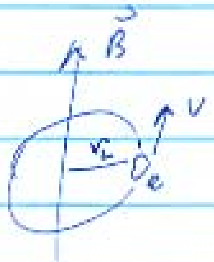
$$\left. \begin{aligned} r_i &= 10^6 \text{ km} \\ r_f &= 10 \text{ km} \\ B_i &= 10^{-2} \text{ T} \end{aligned} \right\} \Rightarrow B_f = 10^8 \text{ T}$$

• Magnetars

- Boost of $\vec{B}$-field by another factor of 1,000 by dynamo action. $\Rightarrow \vec{B} = 10^{11}$ T

Note: This is larger than "quantum-critical field," when QED effects kick in!

Consider: motion of electron in $\vec{B}$-field



$$e v B = \frac{m_e v^2}{r_L}$$

$$\Rightarrow \boxed{r_L = \frac{m_e v}{e B}}$$ "Larmor radius"

If $\vec{B}$-field is very strong, we can have

$$r_L = \lambda_{dB} = \frac{h}{p}$$

Consider NR/UR boundary: $v \sim c$, $p \sim m_e c$

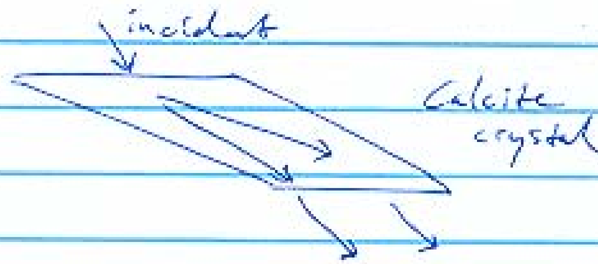
$$\frac{m_e c}{e B_{QED}} = \frac{h}{m_e c}$$

$$\Rightarrow B_{QED} = \frac{m_e^2 c^2}{e \hbar}$$

$$= 4 \times 10^9 \, T$$

Thus: $B_{magnetar} > B_{QED}$.

→ weird effects



(1) Vacuum birefringence

(2) Photon splitting

→ Distort shape of atoms
(e.g. Fe in NS crust)

distort into spindle