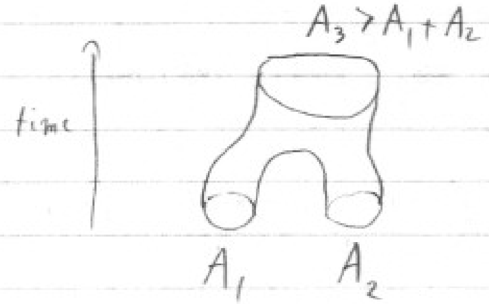
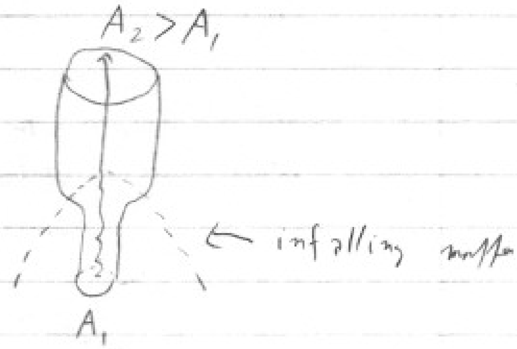
# Lecture 15: Hawking Radiation

- Area-increase Theorem

– Q: How do BHs evolve with time?

– Hawking's B/G idea:



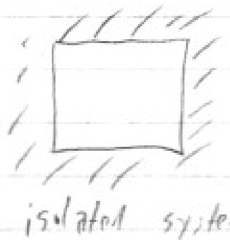




→ Area of BH event horizon (almost) always increases

(or stays constant): $\boxed{\dfrac{dA_{BH}}{dt} \geq 0}$

- Brief review of Entropy

– recall: 2nd Law of Thermodynamics




$$\frac{dS}{dt} \geq 0$$

→ notice: curious similarity to BH area theorem!

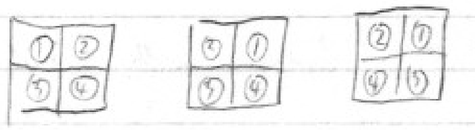
– Entropy: 2 definitions

(i) thermodynamics: $dS \equiv \frac{dQ}{T}$ ← Heat

(ii) statistical mechanics: $S = k_B \ln W$

(Boltzmann's epitaph)

→ $W$ = number of microstates giving rise to same macrostate



- <u>BH Entropy</u>

→ Jacob Bekenstein (1972): exploit curious similarity!

~ Postulate: $$\boxed{S_{BH} = k_B \frac{1}{4} \frac{A_{BH}}{A_{Pl}}}$$ → "BH thermodynamics"

→ "Horizon area IS entropy!"

$$A_{BH} = 4\pi R_s^2$$

Planck area: $A_{Pl} = l_{Pl}^2 = \frac{G\hbar}{c^3}$

↑

Planck length: $l_{Pl} = c\, t_{Pl} \approx 10^{-33}$ cm

$$\Rightarrow \boxed{S_{BH} = k_B \frac{4\pi G}{\hbar c} M^2}$$

→ <u>Problem</u>: If BHs have entropy, they also must have non-zero temperature!

→ Use: $T_{BH}\, dS_{BH} = dQ_{BH} = d(Mc^2) = c^2\, dM$

(7)

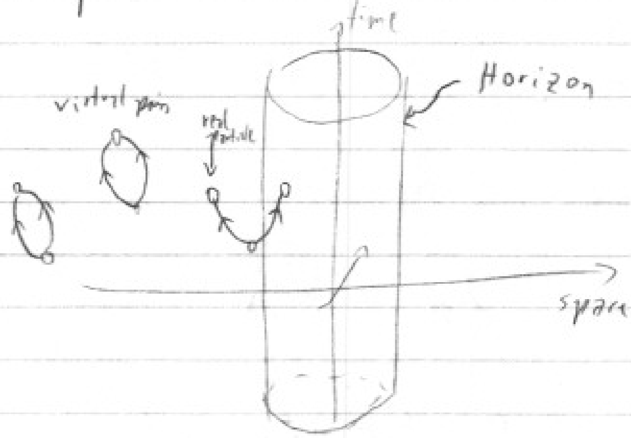
$$dS_{BH} = k_B \frac{8\pi G}{\hbar c} M\, dM$$

$$\Rightarrow \boxed{T_{BH} \overset{?}{=} \frac{\hbar c^3}{8\pi G k_B} M^{-1} \simeq 10^{-7}\,K \left(\frac{M}{M_\odot}\right)^{-1}}$$

→ BIG problem: Any body with $T \neq 0$ radiates / emits photons. But, BH were not supposed to let anything escape !!

• Hawking Effect

→ consider pair-creation near event horizon!




– Energy conservation: real particle (outside horizon) → positive energy $\varepsilon_1 = \varepsilon > 0$

partner " (inside ") → negative energy $\varepsilon_2 = -\varepsilon < 0$

- create both particles + photons
- for photons: black-body radiation (→ thermal equilibrium)

$$\Rightarrow \text{photon density: } n_\gamma = \underbrace{\frac{\pi^2 k_B^3}{15 \hbar^3 c^3}}_{\text{const.}} T^3$$

- estimate mean distance between photons

$$l_{\gamma\gamma} \sim n_\gamma^{-1/3} \sim \frac{\hbar c}{k_B T}$$

- equate this with characteristic "size" of BH → also measure of spacetime curvature!

$$l_{\gamma\gamma} \simeq R_s \leftarrow \text{Schwarzschild radius}$$

$$\Rightarrow T \simeq \frac{\hbar c^3}{G k_B} M^{-1} \simeq T_{BH}$$

- estimate "BH luminosity"

$$L_{BH} \simeq 4\pi R_s^2 \sigma_{SB} T_{BH}^4 \quad \text{(black-body law)}$$

[$\sigma_{SB}$ = Stefan-Boltzmann constant = $5.67\times10^{-5}$ erg s$^{-1}$ cm$^{-2}$ K$^{-4}$]

$$\simeq 10^{-20} \text{ erg s}^{-1} \left(\frac{M}{M_\odot}\right)^{-2}$$

78

- BH Evaporation

→ estimate: How long does it take to completely evaporate a BH? → "Hawking time" $\tau_{Hawk}$

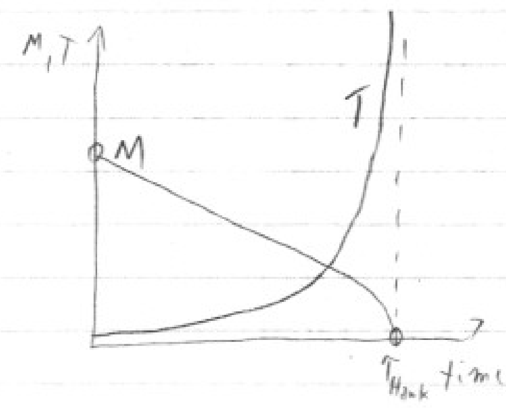
$$\frac{d(Mc^2)}{dt} = -L_{BH} = -10^{-20}\ \text{erg s}^{-1} \left(\frac{M}{M_\odot}\right)^{-2}$$

→ Solve by separation of variables:

$$\boxed{\tau_{Hawk} \simeq 10^{66}\ \text{yr} \left(\frac{M(t=0)}{M_\odot}\right)^3}$$

→ Notice: "Normal" (solar mass) BH will take forever to evaporate

But: eventually this will happen




→ Ask: BH mass, small enough to evaporate within age of the universe (Hubble time $\tau_H = 13.7$ Gyr)?

⇒ Find: $M \sim 10^{14}$ g

→ "mini BHs", possibly created briefly after Big Bang

# Information Content of BHs

- calculate BH entropy

$$S_{BH} \simeq k_B\, 10^{77} \left(\frac{M}{M_\odot}\right)^2$$

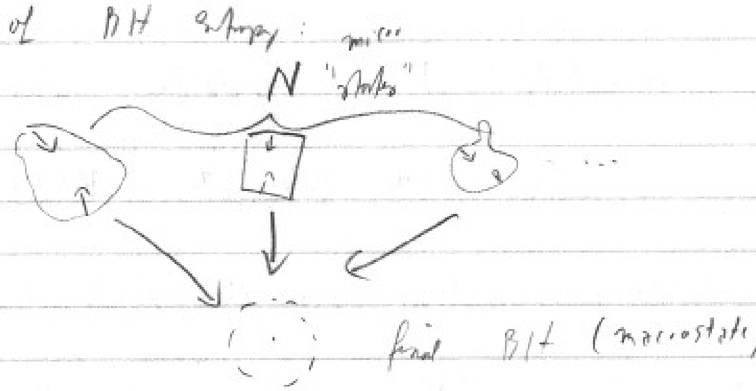
→ This is HUGE! (cf. Sun's entropy: $S_\odot \simeq k_B\, 10^{58}$)

- Q: Why is BH entropy so large?

→ A: connected to 'No-Hair-theorem'

- Interpretation of BH entropy:

Pre-collapse stars




- No-Hair theorem: Almost all pre-collapse stars (microstates) will end up in SAME BH (macrostate)

$$k_B 10^{77} \sim S = k_B \ln N$$

$$\Rightarrow N = \exp[10^{77}] \sim 10^{10^{77}}$$