HW #2

$\boxed{1}$

a) Using $P_c = \dfrac{GM^2}{R^4}$, we have

$$P_c = \frac{GM^2}{(0.01\ R_\odot)^4}\left(\frac{M}{M_\odot}\right)^{4/3}$$

$$= \underbrace{\frac{G\ M_\odot^2}{(0.01\ R_\odot)^4}}_{K_1}\left(\frac{M}{M_\odot}\right)^{\overbrace{10/3}^{x_1}}$$

$$\Longrightarrow \boxed{\begin{aligned} x_1 &= \frac{10}{3} \\ K_1 &= 1.2\times10^{23}\ \mathrm{Pa}\end{aligned}}$$

b) Equating the expressions for central pressure,

$$K_{NR}\left(\rho_c / 2m_H\right)^{5/3} = K_1\left(\frac{M}{M_\odot}\right)^{10/3}.$$

Now we solve for ρ_c :

$$\rho_c = \underbrace{2 m_H \left(\frac{K_1}{K_{NR}}\right)^{3/5}}_{K_2} \left(\frac{M}{M_\odot}\right)^{\overbrace{2}^{x_2}}$$

$$\Rightarrow \boxed{x_2 = 2}$$

$$K_2 = 2 m_H \left(\frac{K_1}{K_{NR}}\right)^{3/5}$$

$$= 8.9 \times 10^{9} \text{ kg m}^{-3},$$

where $K_{NR} = 2.3 \times 10^{-38}$ Pa·m^5, using the electron mass as m_e.

c) We thus find $M_{crit} = 0.34\ M_\odot$.

$\boxed{2}$

a) We have $\rho = \rho_p$, when

$$\left(\frac{M}{M_\odot}\right)^2 = 10^{-5}$$

$$\Longrightarrow \boxed{M_p = 3 \times 10^{-3} M_\odot}$$

b) The mass of Jupiter is $\sim 10^{-3} M_\odot$, which is very close to M_p. Thus, Jupiter is likely partially supported by degeneracy pressure, in addition to electrostatic forces.

3 We have the Compton wavelength as

$$\lambda_c = \frac{h}{m_o c},$$

for a particle of mass m_o. The Compton wavelength corresponding to the Planck mass, $\left(\frac{hc}{G}\right)^{1/2}$, is thus

$$\lambda_c = \left(\frac{hG}{c^3}\right)^{1/2},$$

which defines the Planck length, $l_{pl} \equiv \left(\frac{hG}{c^3}\right)^{1/2}$.

In meters,

$$\boxed{l_{pl} = 4 \times 10^{-35} \text{ m}},$$

and it defines the size of the Schwarzschild radius around the smallest black describable by GR.