Feb. 7, 2008

White Dwarfs cont'd

Chandrasekhar mass : $M_{ch} \simeq \frac{1}{4}\left(\frac{3}{8}\right)^{3/2}\left(\frac{hc}{G}\right)^{3/2}\frac{1}{m_H^2}$

$= 1.7\ M_\odot$

$\boxed{\rightarrow \text{Really, } \sim 1.4\ M_\odot}$

- Rephrase in terms of "Planck mass"

$$M_{pl} \equiv \left(\frac{hc}{G}\right)^{1/2}$$

$$= 5 \times 10^{-8}\ \text{kg}$$

$$= 10^{19}\ \frac{\text{GeV}}{c^2}$$

- Meaning: smallest BH mass that can be described by GR:

(GR)

$$R_s \gtrsim \lambda_c$$

Compton wavelength (QM)

Or, $$\frac{GM_{BH}}{c^2} \gtrsim \frac{h}{m_{BH}c}$$

$$\Longrightarrow \quad M_{BH,\,min} = \underbrace{\left(\frac{hc}{G}\right)^{1/2}}_{M_{pl}}$$

At Planck scale, need quantum gravity (QM + GR) to describe nature.

Now,
$$M_{Ch} = \frac{1}{4}\left(\frac{3}{8}\right)^{3/2} \frac{m_{pl}^3}{m_H^2} \sim \frac{m_{pl}^3}{m_H^2}$$

→ Fundamental concept in astrophysics

- Re-phrase in terms of fundamental strength of gravity

- <u>recall</u>: How to measure strength of e-m interaction

<u>2 protons</u>

→ <u>Potential energy</u>:

$$U_{em} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r},$$

where $\epsilon_0 = 8.9 \times 10^{-12}\, C^2 N^{-1} m^{-2}$

→ Evaluate at $r = \lambda_{c,\text{proton}} = \dfrac{h}{m_H c}$

$$\simeq \frac{\hbar}{c m_H}$$

→ define e-m "fine structure constant":

$$\alpha \equiv \frac{U_{em}(r=\lambda_c)}{m_H c^2}$$

$$= \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

$$= \frac{1}{137}$$

→ By analogy, define the "gravitational fine structure constant":

$$\alpha_G = \frac{U_g(r=\lambda_c)}{m_H c^2} = \frac{G m_H^2}{\hbar c}$$

$$= 10^{-38} \leftarrow \text{weak!}$$

Now:

$$M_{ch} \simeq \left(\frac{\hbar c}{G}\right)^{3/2} \frac{1}{m_H^2}$$

$$\simeq \alpha_G^{-3/2} m_H \quad = 10^{57} m_H \sim 1 M_\odot$$

<u>Note</u>: Since gravity is so weak, need 10^{57} nucleons to overwhelm e-m force

$\Rightarrow M_{ch} \sim 1 M_\odot$ is typical mass of

any star!

- <u>Cooling of WDs</u>

- recall $$\boxed{L = 4\pi R^2 \sigma_{SB} T_{eff}^4}$$

"Stefan-Boltzmann law"

$$\sigma_{SB} = 5.67 \times 10^{-8} \ W m^{-2} K^{-4}$$

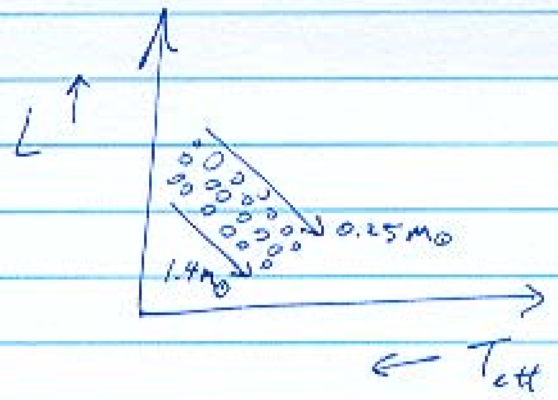
$\rightarrow$ mass-radius relation for NR WD

$$R = 0.01 \ R_\odot \left(\frac{M}{M_\odot}\right)^{-1/3}$$

$\rightarrow$ from observations, typically $T_{eff} \simeq 20{,}000$ K

$$\Rightarrow \boxed{L \simeq 10^{-2} L_\odot \left(\frac{M}{M_\odot}\right)^{-2/3} \left(\frac{T}{20{,}000 \ K}\right)^4}$$

→ WD cooling tracks on Hertzsprung - Russell diagram



Q: How long does it take for WD to radiate away all of its thermal energy?

A: cooling timescale

$$\boxed{\tau_{cool} = \frac{E_{th}}{L}}$$

→ $L \sim 10^{-2} L_\odot$

→ $E_{th} \hat{=}$ thermal energy $= \frac{\frac{3}{2} k_B T}{12 m_H} M$

→ use $T \sim 10^8$ K, $M \sim M_\odot$

$\Rightarrow E_{th} \approx 10^{41}$ J

$$\boxed{\Rightarrow \tau_{cool} = \text{few} \times 10^9 \text{ yr}}$$

→ comparable to the age of the Universe, 13 Gyr