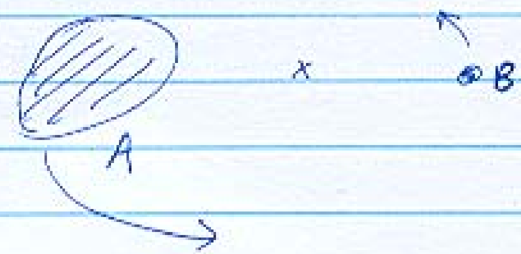
Feb. 5, 2008

White Dwarfs

- **Discovery**: by observation (19th century)

  - Sirius: double-star system



  - Sirius B: weird properties

$$\left.\begin{array}{l} M \sim 1\, M_\odot \\ R \sim \frac{1}{100} R_\odot \end{array}\right\} \Rightarrow \langle \rho \rangle \sim 10^9\ \mathrm{kg\,m^{-3}}$$

(while $\langle \rho \rangle_\odot \sim 10^3\ \mathrm{kg\,m^{-3}}$)

→ typical parameters: for white dwarfs (WDs), small (dwarfs) and hot (white)

- Estimate strength of GR effects

- important concept in general relativity (GR)
  → Schwarzschild radius → $R_s \equiv \frac{2GM}{c^2}$.

<u>intuition</u>: consider escape velocity $\left\{ \begin{array}{l} E \to 0, \\ \text{as } r \to \infty \end{array} \right\}$

$$E = \frac{1}{2} m v^2 - \frac{GMm}{r}$$

$$\Rightarrow v_{esc}^2 = \frac{2GM}{R}$$

$$\to \text{For } v_{esc} = c \implies R = R_s = \frac{2GM}{c^2}.$$

$$\left(R_{s\odot} = 3\,\text{km} \right)$$

<u>real meaning</u>: $\underbrace{\text{circumference}}$ of black hole,

event horizon

i.e. $C_{BH} = 2\pi R_s$

<u>Strength of GR effects</u>

(i) Sun : $\frac{R_s}{R} \sim 10^{-6}$

(ii) WD : $\frac{R_s}{R} \sim 10^{-4}$

$\Bigg\}$ GR not important

(iii) NS: $\frac{R_s}{R} \sim 0.5 \rightarrow$ GR very important

• Basic properties of WDs

- for WD to be in hydrostatic equilibrium, need

$$\langle P \rangle = \frac{-1}{3}\frac{E_{pot}}{V} = \frac{1}{3}\frac{GM^2}{R^4}$$

- If mass is not too large, WD pressure due to NR degeneracy pressure from electrons,

$$\langle P \rangle = K_{NR} \langle n_e \rangle^{5/3}$$

- Here, $\langle n_e \rangle \hat{=}$ average density of free electrons

Q: How is this related to mass density?

A:

- typical WD matter: ${}^{4}_{2}He$, ${}^{12}_{6}C$, ${}^{16}_{8}O$

→ 2 nucleons (proton or neutron) per free electron

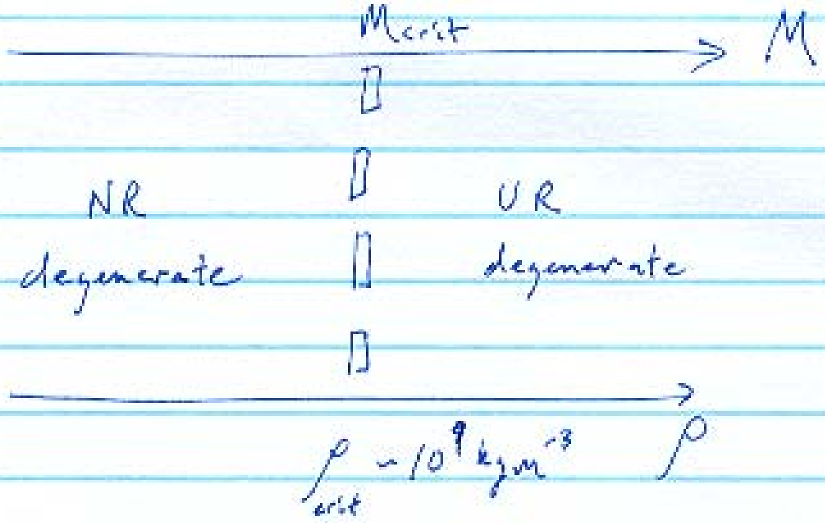
$$\Rightarrow \boxed{\rho = 2 m_H n_e}$$

Thus,

$$\frac{K_{NR}}{(2 m_H)^{5/3}} \langle \rho \rangle^{5/3} = \frac{1}{3} \frac{G M^2}{R^4}$$

$$\Rightarrow \langle \rho \rangle^{5/3} \propto \frac{M^2}{R^4}$$, and with $\langle \rho \rangle \sim \frac{M}{R^3}$

$$\boxed{\langle \rho \rangle \propto M^2}$$

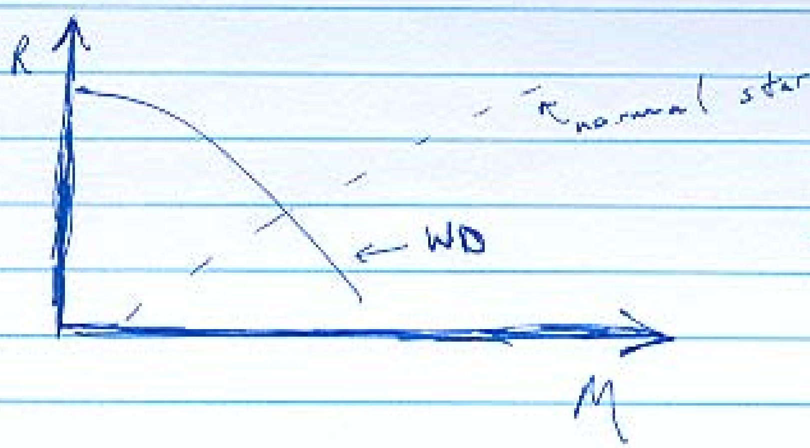


For $M < M_{crit}$ (NR conditions)

$$\frac{M}{R^3} \sim \langle \rho \rangle \propto M^2$$

$$\Rightarrow R \propto M^{-1/3} \quad \text{or} \quad R = 0.01 \, R_\odot \left(\frac{M}{M_\odot} \right)^{-1/3}$$

"Mass-radius relation for white dwarfs"



$\Rightarrow$ Very strange!

- Properties of UR white dwarfs

$$M \rightarrow M_{crit} \quad \Longrightarrow \quad \langle P \rangle \rightarrow \langle P_{UR} \rangle$$

$$\langle P_{UR} \rangle = \frac{K_{UR}}{(2m_H)^{4/3}} \langle \rho \rangle^{4/3} \quad ; \quad K_{UR} = \frac{hc}{4}\left(\frac{3}{8\pi}\right)^{1/3}$$

$$\simeq \frac{hc}{8}$$

$$\langle P_{UR} \rangle \geq \frac{1}{3}\frac{GM^2}{R^4}$$, to counteract gravity.

- Use $\langle \rho \rangle \sim \frac{M}{R^3}$

$$\Longrightarrow \frac{K_{UR}}{(2m_H)^{4/3}} \frac{M^{4/3}}{R^4} \gtrsim \frac{1}{3}\frac{GM^2}{R^4}$$ (radius cancels out!)

$$\Longrightarrow M \lesssim \underbrace{\left(\frac{3}{8}\right)^{3/2}\left(\frac{hc}{G}\right)^{3/2}\frac{1}{(2m_H)^2}}_{M_{ch}}$$

- Chandrasekhar limit (or mass):

$$\boxed{M_{ch} = \frac{1}{4}\left(\frac{3}{8}\right)^{3/2}\left(\frac{hc}{G}\right)^{3/2}\frac{1}{m_H^2} = 1.7\ M_\odot}$$

precise calculation $\Longrightarrow$ $M_{ch} = 1.4\ M_\odot$

$\rightarrow$ No WD can exist with $M > M_{ch}$!