Feb. 19, 2008

Intro. to General Relativity (cont'd)

Q: How can we connect gravity and spacetime geometry?

$\Delta\tau \equiv \Delta\tau_{em} \hat{=}$ proper time measured by clock / observer at radius r where we emit photons

$\Delta\tau_{\infty} \equiv \Delta t \hat{=}$ co-ordinate time
→ time measured by clock / observer at large distance

$$\Delta\tau = \left(1 - \frac{GM}{rc^2}\right)\Delta t$$

→ connect to spacetime interval

$$ds^2 = -c^2 d\tau^2 = -c^2 \left(1 - \frac{GM}{rc^2}\right)^2 dt^2$$

$(dx = dy = dz = 0$ for clock at rest$)$

→ We have considered weak fields:

$\frac{GM}{rc^2} \ll 1$; e.g., in Solar System

$$\left(1 - \frac{GM}{rc^2}\right)^2 \simeq 1 - \frac{2GM}{rc^2}$$

Taylor expand

⇒ Spacetime interval for Newtonian gravity

$$ds^2 = -c^2 \left(1 - \frac{2GM}{rc^2}\right) dt^2 + \underbrace{dx^2 + dy^2 + dz^2}_{\text{Usual Euclidean space}}$$

→ Recall Newtonian potential: $\varphi = \frac{-GM}{r}$

gravity ⟷ geometry (metric coefficients)

$$\varphi \longleftrightarrow 1 + \frac{2\varphi}{c^2}$$

→ In general, $ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$

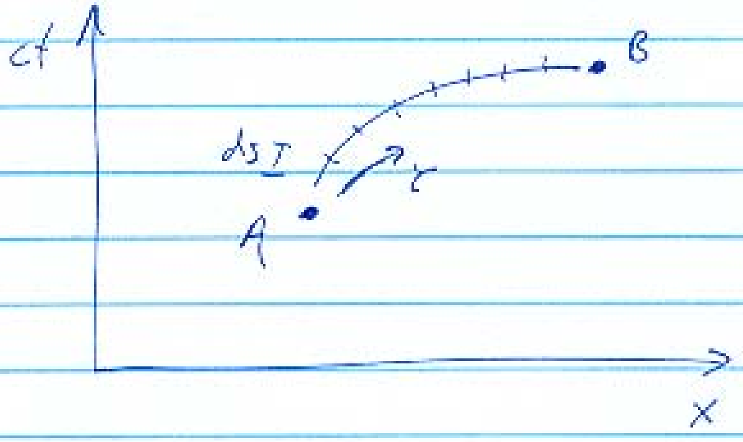
$g_{\mu\nu} \equiv$ "metric tensor" → 4×4 matrix

e.g., for weak field

$\rightarrow g_{00} = -\left(1 + \frac{2\Psi}{c^2}\right)$

• <u>Motion of particles in GR</u>

- <u>Rule</u>: particles move along "straightest" lines ("geodesics") through spacetime
  → applies only to freely-falling particles



- Find worldline

$$x^\mu(\tau) = \begin{pmatrix} t(\tau) \\ x(\tau) \\ y(\tau) \\ z(\tau) \end{pmatrix}$$

that extremizes path

$$\int_A^B ds = \text{extremum}$$
(min. or max.)

Solve by calculus of variations!

$$\Rightarrow \frac{d^2 x^\mu}{d\tau^2} + \left(\begin{array}{l}\text{terms with } 1^{st} \\ \text{derivatives of } g_{\mu\nu}\end{array}\right) = 0$$

→ equation of motion in GR

("Curved space tells matter how to move")

- <u>Motion of photons</u>

→ consider photons moving along x-direction in SR $(dy = dz = 0)$

$$dx = c\,dt$$

$$\Rightarrow ds^2 = -c^2 dt^2 + dx^2 = 0$$

$$\Rightarrow -c^2 d\tau^2 = 0$$ → Photons move along 'null geodesics'