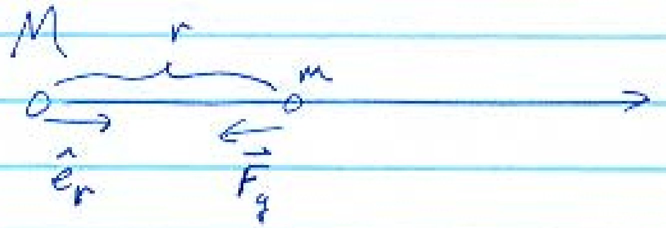
Feb. 12, 2008

General Relativity: Very brief intro.

- What is general relativity (GR)?
  - gravity + special relativity (SR) = GR
  - $\boxed{\text{Basic idea of GR: gravity is curved spacetime!}}$

- Recall basics of Newtonian gravity
  - consider two particles



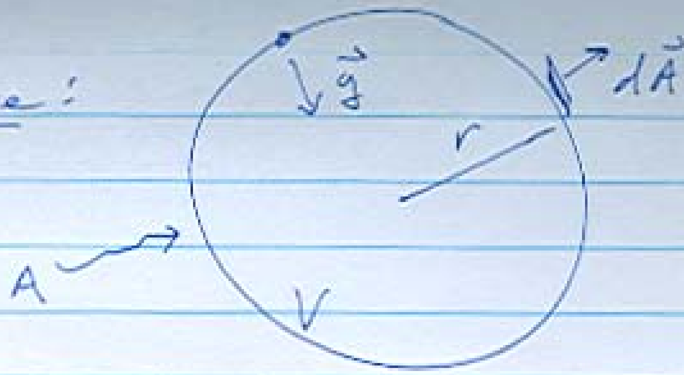
$$\vec{F}_g = -\frac{GMm}{r^2}\hat{e}_r$$

or: $$\vec{g} = \frac{\vec{F}_g}{m}$$

→ gravitational potential ("field")

$$\varphi = \frac{-GM}{r} \Rightarrow \vec{g} = -\nabla\varphi$$

- Re-phrase:



$$\int \nabla \cdot \vec{g}\, dV = \oint_A \vec{g} \cdot d\vec{A} = -4\pi r^2 g$$

$$= -4\pi G \int \rho(r)\, dV$$

$$\Longrightarrow \nabla \cdot \vec{g} = -4\pi G \rho$$

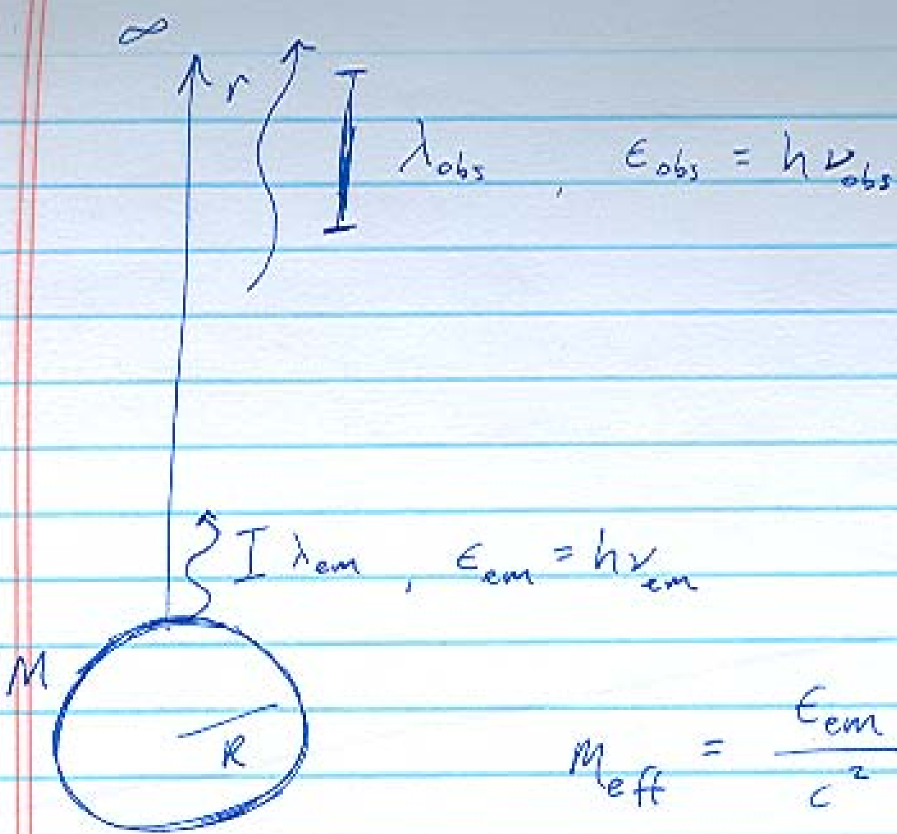
$$\Longrightarrow \boxed{\nabla^2 \varphi = 4\pi G \rho}$$ Poisson equation -

→ "field theory of Newtonian gravity"

$$\boxed{\frac{d^2\vec{r}}{dt^2} = \vec{g} = -\nabla \varphi}$$ "Equation of motion"

Gravitational Redshift

- Use Newtonian theory (no GR)



$$M_{eff} = \frac{\epsilon_{em}}{c^2}$$

- photon has to climb out of potential well

$\rightarrow$ work done:

$$W = -\int_R^\infty \vec{F} \cdot d\vec{r}$$

$$= \frac{GMm_{eff}}{R} = \frac{GM}{R}\frac{\epsilon_{em}}{c^2}$$

$\rightarrow$ $$\epsilon_{obs} = \epsilon_{em} - \frac{GM}{Rc^2}\epsilon_{em}$$

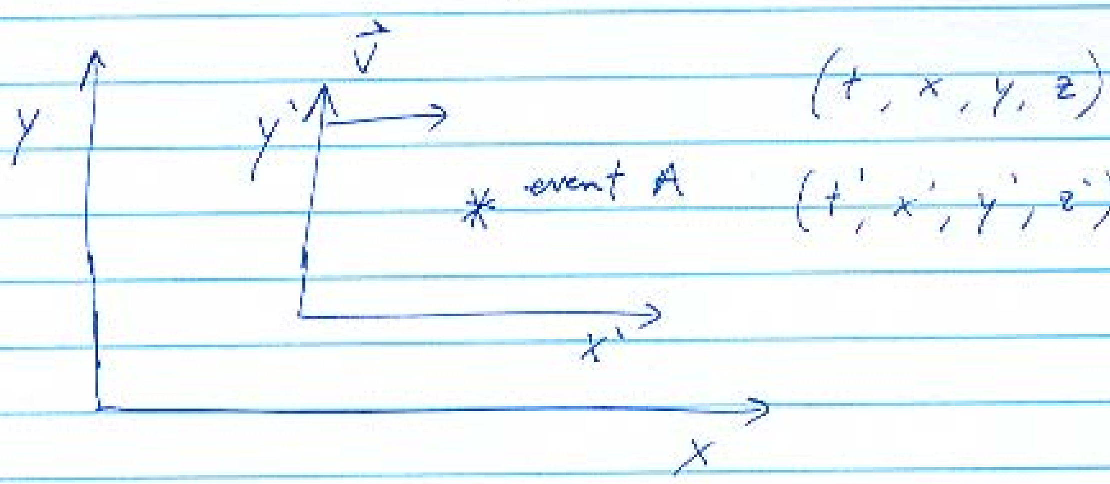
$$= \epsilon_{em}\left(1 - \frac{GM}{Rc^2}\right)$$

$\Rightarrow$ $$\nu_{obs} = \nu_{em}\left(1 - \frac{GM}{Rc^2}\right)$$

- Special Relativity

→ events (t, x, y, z)

→ consider events in 2 different inertial frames (inertial system ; IS)



Lorentz transformations

$$y' = y$$

$$z' = z$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

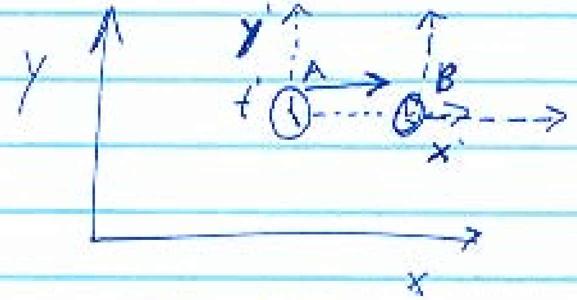
$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- often consider differences in coordinates

$$\Delta x = x_B - x_A \longrightarrow dx$$

$$\Delta t = t_B - t_A \longrightarrow dt$$

- Example: time dilation



- in clock rest frame,

$$\Delta x' = \Delta y' = \Delta z' = 0$$

t ①

$$\Delta x = v \Delta t$$

$$\Delta t' = \frac{\Delta t - \frac{v}{c^2}\Delta x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{\Delta t - \frac{v^2}{c^2}\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} = \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$

$\Rightarrow$ "Moving clocks run slower"!

in SR time + space are relative

Q: What then is real (absolute)?