$\boxed{1}$

a) The highest mass possible of a neutron star

b) Radiation emitted from the vicinity of a black hole when a virtual photon is absorbed by the BH and its pair escapes.

c) Light can escape the event horizon, even if it must return before escaping to infinity, for the Dark Star. No light can escape the event horizon in GR.

d) Entropy is a general measure of disorder, in some sense. Formally, $dS = \frac{dQ}{T}$ and

$$S = k_B \ln W,$$

e) Singularities are always surrounded by event horizons.

f) The time after recombination and before the first stars form.

g) Cold dark matter has structure form from 'bottom-up', while hot DM has 'top-down'

$\longrightarrow$ Cold dark matter is more likely correct!

h) The entropy and area always increase, barring Hawking Radiation, too, $S = k_B \frac{A_{BH}}{A_{Pl}}$.

i) 10 km

j) Black holes are always perfectly spherical, and retain no memory of the objects that fell into them.

2

a) Primordial gas lacks cooling agents. H_2 only cools gas to ~200 K, a much higher temp. than in present day S.F.
⟹ The mass of gravitationally unstable gas clouds is thus higher.

b) They contain only metals from a few or one SN explosion, which means they were preceded by only one or a few generations of stars after the big bang.
We learn about the types of stars that existed in the early universe and the conditions for star formation

c) $t_{ff} \sim \sqrt{\frac{1}{G\rho}}$

$\sim 2 \times 10^{-4}$ sec

$\longrightarrow$ NSs collapse quickly if they lose pressure support!

3

a) $ds^2 = -c^2 dt^2 \left(1 - \frac{R_s}{r}\right) + \frac{dr^2}{\left(1 - \frac{R_s}{r}\right)} + d\Omega^2$

b) ϕ and θ are the regular angular coords. r is the coordinate distance from the singularity. t is the coordinate time.

c) 3×10^6 km

4

a) $E_{grav} \sim \frac{-GM^2}{R} \sim -10^{45}$ J

b) $P_{rad} \sim \frac{1}{3} a_{rad} T^4$

$$\sim 10^{16} \ J m^{-3}$$

$$P_{gas} \sim \frac{\rho k_B}{m_H} T$$

$\sim 10^{15} \ J m^{-3}$, where $\rho = \frac{M}{R^3}$.

$\Rightarrow$ P_{rad} is considerably larger than P_{gas}.

$\boxed{5}$ Separation of variables gives

$$r^{1/2} dr = -c R_s^{1/2} d\tau$$

$$\Rightarrow \Delta\tau \left(-c R_s^{1/2}\right) = \int_{4R_s}^{R_s} r^{1/2} dr$$

$$\Rightarrow \boxed{\Delta\tau = \frac{14}{3} \frac{R_s}{c}}$$

6

a) We have that $T_{BH} = 10^{-7}\left(\frac{M_\odot}{M}\right)$ K

and that $R_s = \frac{2GM}{c^2}$. Thus,

using the given formula, we obtain

$$4\pi\left(\frac{2GM}{c^2}\right)^2 \sigma \left(10^{-7}\frac{M_\odot}{M}\right)^4 = L_\odot$$

$$\Rightarrow \boxed{M = 2.5 \times 10^{3} \text{ kg}}$$

b) $S_{BH} \sim k_B \, 10^{77} \left(\frac{M}{M_\odot}\right)^2$

$\sim 1.6 \times 10^{23} \, k_B$

$\boxed{7}$

a) (i) $\frac{d}{dt}\frac{\partial L}{\partial \dot{t}} = \frac{\partial L}{\partial t}$. So, since $\frac{\partial L}{\partial t} = 0$,

we have $\frac{\partial L}{\partial \dot{t}}$ = constant and

$$\frac{\partial L}{\partial \dot{t}} = 2c^2 \dot{t} \implies \dot{t} = \text{constant}$$

This is conservation of energy.

(ii) $\frac{\partial L}{\partial \dot{\phi}}$ = constant, here, similarly. Thus,

$$\frac{\partial L}{\partial \dot{\phi}} = -2r^2\dot{\phi} = \text{constant}, \text{ or}$$

$$r^2\dot{\phi} = \text{constant},$$

This is conservation of angular momentum.

b) We have $\frac{\partial L}{\partial r} = -2\dot{\phi}^2 r$, $\frac{\partial L}{\partial \dot{r}} = -2\dot{r}$,

and so $\frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = -2\ddot{r}$ and $-2\ddot{r} = -2\dot{\phi}^2 r$.

$\implies \ddot{r} = r\dot{\phi}^2$, which is centripetal force.