$\boxed{1}$

(a) The free-fall time is the time it takes for matter to collapse to a point, under only the influence of gravity.

(b) The Planck mass is that of the smallest black hole that can be described by General Relativity on its own.

(c) Pressure is "momentum flux," the amount of momentum that moving particles carry across a surface in a given time.

(d) A gas becomes degenerate under conditions of sufficiently high density and low temperature: $n \gtrsim \left(\frac{h}{p}\right)^{-1/3}$,

when $p \hat{=}$ momentum of a single particle.

(e) A degenerate gas becomes relativistic at very high density, when $v \to c$ for the constituent particles.

(f) Fusion of hydrogen to helium is the energy source for main-sequence stars.

(g) $R_{WD} \sim 10{,}000$ km

$R_{\odot} \sim 7 \times 10^5$ km

(h) The Chandrasekhar mass is the maximum mass that a white dwarf may have without collapsing.

(i) The spacetime interval is the measure over a spacetime that is invariant; that is, every observer in an inertial frame of reference will measure the same spacetime interval between two events.

(j) The gravitational fine-structure constant, $\alpha_G = \frac{G m_H^2}{\hbar c}$, is the ratio of gravitational potential energy of a proton to its rest mass energy, while

3

the electromagnetic fine-structure constant is the ratio of electrostatic potential energy of a proton to its rest mass energy.

$\boxed{2}$

(a) Using $t_{ff} = \sqrt{\frac{1}{G\rho}}$, and $\rho_\odot = 1{,}400 \text{ kg m}^{-3}$ for the Sun, and $\rho_{WD} = 1{,}000\, \rho_\odot$ for the white dwarf, we have

$$t_{ff_\odot} \simeq 1 \text{ hour},$$

while

$$t_{ff_{WD}} \simeq 3 \text{ seconds.}$$

(b) This union is termed space-time.

(c) Photons travel at the speed of light, c, so the time that they measure is always zero because $ds^2 = 0$. Time dilation means that "moving clocks move slower," while clocks traveling at c completely stop!

$\boxed{3}$

(a) For these boundary conditions, the integral for the pressure as a function of radius, r, is

$$\int_0^{P(r)} dP = -\frac{4\pi G}{3}\rho_c^2 \int_R^r r \exp\left(\frac{-r^2}{a^2}\right) dr$$

$$= \frac{2\pi G}{3}\rho_c^2 a^2 \left[\exp\left(\frac{-r^2}{a^2}\right) - \exp\left(\frac{-R^2}{a^2}\right)\right]$$

The central pressure, $P_c = P(r=0)$:

$$P_c = \frac{2\pi G}{3}\rho_c^2 a^2 \left[1 - \exp\left(\frac{-R^2}{a^2}\right)\right]$$

(b) Differentiate dP/dr to find the minimum:

$$\frac{d}{dr}\left(\frac{dP}{dr}\right) = \frac{-4\pi}{3} G\rho_c^2 \left[\exp\left(\frac{-r^2}{a^2}\right) - \frac{2r^2}{a^2}\exp\left(\frac{-r^2}{a^2}\right)\right]$$

$$= \frac{-4\pi}{3} G\rho_c^2 \exp\left(\frac{-r^2}{a^2}\right)\left[1 - \frac{2r^2}{a^2}\right]$$

2

The minimum of dP/dr will occur where $d^2P/dr^2 = 0$, which occurs at

$$1 - \frac{2r_{min}^2}{a^2} = 0$$

$$\Rightarrow \boxed{r_{min} = \frac{a}{\sqrt{2}}}.$$

4

(a) Equating the two expressions for p_F that are given, we obtain

$$m_e c = h \left(\frac{3}{8\pi}\right)^{1/3} n_{crit}^{1/3}$$

$$\Rightarrow \boxed{\begin{aligned} n_{crit} &= \frac{8\pi}{3}\left(\frac{m_e c}{h}\right)^3 \\ &= 5.7 \times 10^{35}\ m^{-3} \end{aligned}}$$

(b) White dwarfs are composed primarily of elements for which the number of neutrons is roughly equal to the number of protons, such as C^{12} and O^{16}. Then, because the number of electrons is equal to the number of protons, the mass density, contributed by neutrons and protons, is

$$\rho = 2 m_H n_e .$$

Thus, we obtain $\boxed{\rho_{crit} \simeq 2 \times 10^9\ kg\ m^{-3}}$.

$\boxed{5}$ Using the formulae for D and Δs^2 that are given obtains, with $\Delta t = \Delta y = \Delta z = 0$,

$$D = \sqrt{\Delta s^2}$$

$$= \sqrt{\left(1 + \frac{2GM}{rc^2}\right)} \, \Delta X .$$

For $M = 100 \, M_\odot$ and $r = 2 \times 10^9 \, m$,

$$D = 1.007 \, \Delta X$$

$$= 1.007 \times 10^5 \, m$$

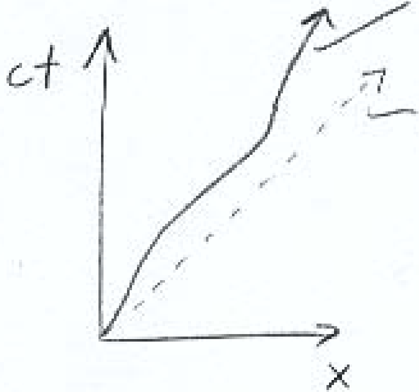
(b) The spacetime in which the measurement is made is not completely free of curvature; although the curvature of spacetime due to the gravity of the star is small, its presence comes through in our measurement.

$\boxed{6}$

(a) The Minkowski metric gives for the spacetime interval

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

(b)



particle – $c^2 t^2 > x^2 \Rightarrow ds^2 < 0$

photon – $c^2 t^2 = x^2 \Rightarrow ds^2 = 0$