3/29/07

Black Holes

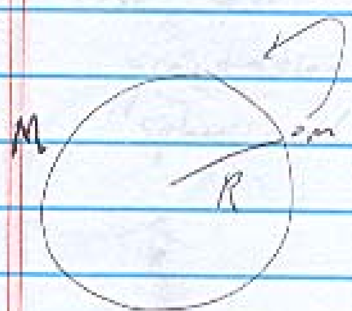
- Early speculations. "Dark Star"

In late 18^{th} century (Enlightenment)
- John Michell (1783)
- Pierre Simon Laplace (Le Système du Monde, 1796)

→ 2 ingredients

(1) Newtonian gravity

(2) Corpuscular theory of light (also Newtonian)



$$E_{tot} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

For no escape, need $E_{tot} < 0$

$\Rightarrow v > v_{esc} = \sqrt{\frac{2GM}{R}}$, in order for particle to escape

For light, $v_{esc} = c$, a critical radius:

$$R_s = \frac{2GM}{c^2},$$

later called the "Schwarzschild radius"

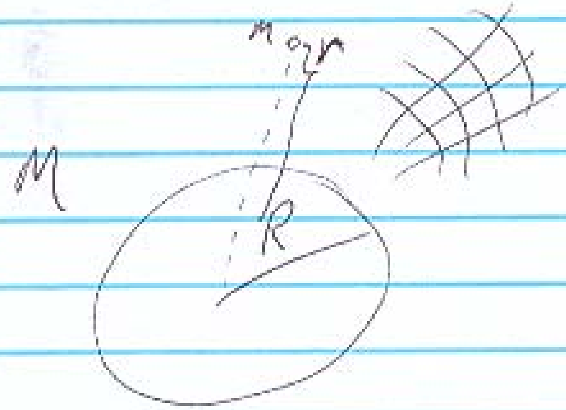
- Crucial difference between dark stars and modern black holes

→ For the dark star, close-in observers would still be able to see the object

→ For modern black holes, photons cannot travel beyond $r > R_S$

<u>Schwarzschild Geometry</u>

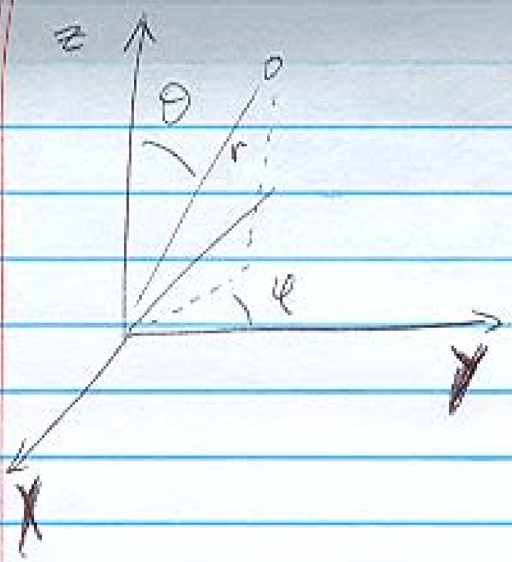
- Use GR to consider geometry (or gravitational field) outside of a spherically symmetric object



← geometry, metric coefficients

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

- Choose, for convenience, spherical coordinates (r, θ, ϕ), reflecting the spherical symmetry



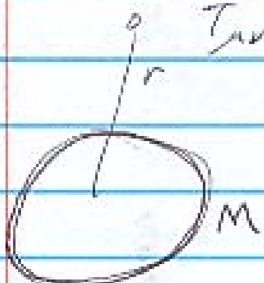
- Spacetime must have general form

$$ds^2 = -A(r)\,c^2 dt^2 + B(r)\,dr^2 + r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right)$$

- Find solution to Einstein field equation

→ <u>outside</u> of the spherical mass distribution ("vacuum solution")

$T_{\mu\nu} = 0$



$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\boxed{G_{\mu\nu}\left(\frac{\partial^2 g_{\mu\nu}}{\partial x^\alpha \partial x^\beta}, \frac{\partial g_{\mu\nu}}{\partial x^\alpha}, g_{\mu\nu}\right) = 0}$$

→ Find solution

$$A(r) = g_{00} = 1 - \frac{2GM}{c^2 r}$$

$$B(r) = g_{11} = \frac{1}{A(r)}$$

$$\boxed{ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2 dt^2 + \frac{dr^2}{\left(1-\frac{2GM}{c^2 r}\right)} + r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right)}$$

"Schwarzschild metric" (K. Schwarzschild, 1916)

- Interpretation of radial coordinate:

- NOT the physical distance from the center
  → spacetime is curved

- "Circumferential radius"

- Consider the physical (proper) area on a surface with r = const.

$$A = r^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\varphi = 4\pi r^2$$

$$C = 2\pi r$$

Flow of time in Schwarzschild geometry

Consider 2 stationary observers $dr = d\theta = d\varphi = 0$

(A) at $r \to \infty$

(B) at r

- Figure out proper time that each observer would measure, τ

$$-c^2 d\tau^2 = ds^2$$

(A) $r \rightarrow \infty \Rightarrow ds^2 = -c^2 dt^2 = -c^2 d\tau_\infty^2$

Thus, $d\tau_\infty = \underbrace{dt}_{\text{"coordinate time"}}$

(B) $ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 = -c^2 d\tau^2$

$$\Rightarrow d\tau = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2} dt$$

$$\boxed{d\tau = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2} d\tau_\infty}$$

- Time slows down deep inside a gravitational potential well ("gravitational time dilation")

- Effect on light

(B) emits light at frequency $\nu_{em} = \frac{1}{d\tau}$

(A) measures different frequency, $\nu_\infty = \frac{1}{d\tau_\infty}$

$$\Rightarrow \nu_\infty = \nu_{em}\left(1 - \frac{2GM}{c^2 r}\right)^{1/2}$$

("gravitational redshift")

- Photons emitted at $r = R_s$ are redshifted to $\nu \to 0$ at $r \to \infty$. No photons make it to $r \to \infty$!