3/27/07

Neutron Stars (cont'd)

- **Upper mass limit (revisit)**

$M_{NS} < M_{OV} \simeq 5 M_\odot$

→ Was derived by assuming
  - GR → OV equation
  - use extreme equation of state (e.o.s.)
    ρ_0 = constant ("incompressible")

→ In general, for e.o.s. we have

$P = K \rho^\gamma$ (K, γ some constants)

γ: adiabatic index
"resistance to compression"

$\Rightarrow \rho = \left(\frac{P}{K}\right)^{1/\gamma}$

For ρ = constant $\Longrightarrow \gamma \to \infty$

However, real equation of state is in NS is "softer" ("less stiff"), with γ lower

But e.o.s. very uncertain; however, we can estimate it, giving $M_{OV} \simeq 1.5 - 3.0\, M_\odot$

Note: All currently known NS masses (from binary systems)

$$M_{NS} \simeq 1.45\ M_\odot$$

Structure of Cold, Dead Stars

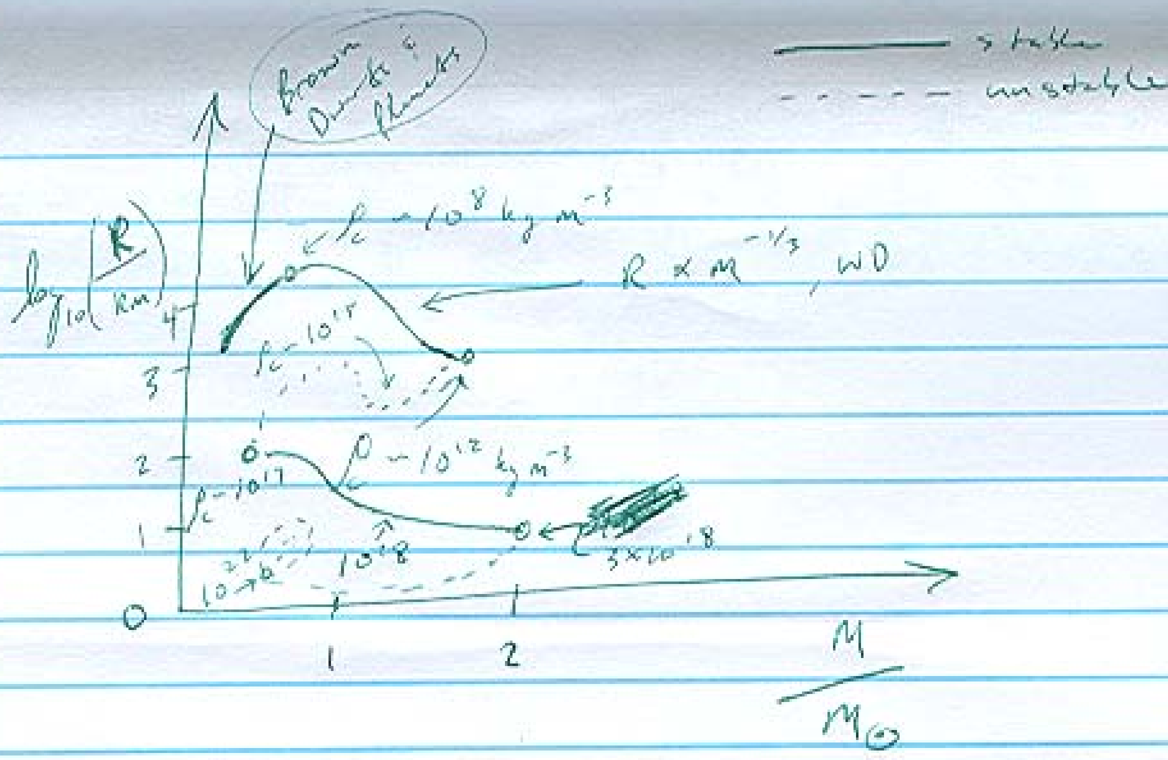
- Construct e.o.s for matter in its 'ground state'
  - → zero energy
  - → All nuclear entirely spent
  - → Cold dead matter ("cold, catalyzed matter")

Construct equilibrium models

- Choose central density, ρ_c
- Calculate central pressure, P_c (from e.o.s.)
- Integrate OV equation $\left(\frac{dP}{dr}\right)$ from $r=0$ to surface, where ρ and $P \simeq 0$, with $r = R$

- Find $M = M(\rho_c)$, $R = R(\rho_c)$

Plot mass-radius relation:

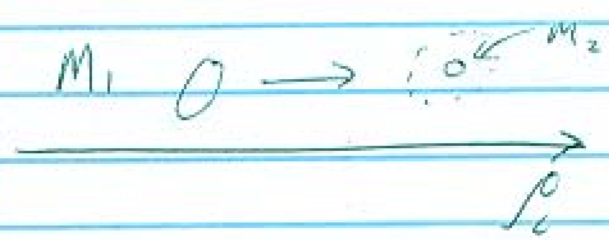


- Stability requires $\dfrac{dM}{d\rho_c} > 0$

Q: Why?

→ Consider case $dM/d\rho_c < 0$

Equilibrium configuration



Estimate for R of cold, dead stars

- Consider WD & NS

	Pressure due to	Mass	Gravity, E_{pot}	Pressure E_{kin}
WD	deg. elec., $n = n_e$	$\rho \simeq 2 m_H n \sim m_H n$	$\frac{G M m_H}{R}$	$m_e c^2$
NS	deg. neutrons $n = n_n$	$\rho = m_H n$	$\frac{G M m_H}{R}$	$m_H c^2$

$$\boxed{m_o c^2 \, ; \, m_o = \begin{cases} m_e & \text{WD} \\ m_H & \text{NS} \end{cases}}$$

- Assume degeneracy pressure at NR/UR boundary

$$\longrightarrow E_{kin} \simeq m_o c^2$$

- For equilibrium, need $E_{pot} \simeq E_{kin}$
  - $M \simeq \rho R^3 \simeq m_H n R^3$ (for both WD & NS)
- Gas is degenerate at NR/UR transition
  - allows us to estimate n

$$l \simeq \lambda_{deB}$$

mean particle separation

deBroglie wavelength

$$\lambda_{deB} \equiv \frac{h}{p} \underset{\text{NR/UR}}{\simeq} \frac{h}{m_0 c} \equiv \lambda_c$$

Compton wavelength

$$(p \simeq m_0 c)$$

$$\Longrightarrow n^{-1/3} \simeq \frac{h}{m_0 c}, \text{ where } m_0 = \begin{cases} m_e & \text{for WD} \\ m_H & \text{for NS} \end{cases}$$

$$\Longrightarrow M \simeq \left(\frac{h}{m_0 c}\right)^{-3} R^3 m_H$$

From $E_{pot} \simeq E_{kin}$,

$$\underbrace{\frac{G m_H^2}{\hbar c}}_{\alpha_G} \left(\frac{h}{m_0 c}\right)^{-3} R^2 \simeq \frac{m_0 c^{\cancel{2}}}{\hbar \cancel{c}} = \frac{m_0 c}{\hbar} \simeq \frac{m_0 c}{h}$$

$$\Longrightarrow \boxed{R \simeq \alpha_G^{-1/2} \frac{h}{m_0 c} = \alpha_G^{-1/2} \lambda_c}$$

<u>Recall</u>: $M_{Ch} = \alpha_G^{-3/2} m_H$, $\alpha_G \sim 10^{-38}$

$$R_{WD} \simeq \alpha_G^{-1/2} \frac{h}{m_e c} \simeq 20{,}000 \text{ km}$$

$$R_{NS} \simeq \alpha_G^{-1/2} \frac{h}{m_H c} \simeq 10 \text{ km}$$

Note: $\frac{R_{WD}}{R_{NS}} \simeq \frac{m_H}{m_e} = 1,836$

Microphysics determines macrophysics!