3/22/07

Neutron Stars (cont'd)

- Neutronization

Basic question: Why are neutron stars made up of neutrons?

Recall: Free neutrons are unstable

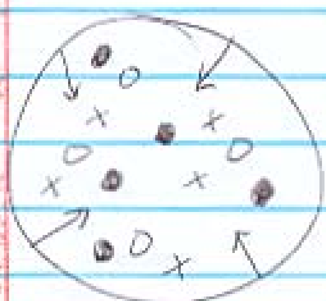
$$n \longrightarrow p^+ + \underbrace{e^- + \bar{\nu}_e}_{\boxed{1.3\ \text{MeV} = (m_n - m_p)c^2}} \qquad (\beta\text{-decay})$$

half-life, $\tau_{1/2} \sim 10$ min.

Why don't neutrons decay in a NS?

→ This would be energetically disfavored!

During the collapse of a massive star's core (en route to NS) n's are bathed in a sea of degenerate, UR electrons!



neutrons - O
protons - ●
electrons - +

Recall: Fermi momentum for degenerate e^-:

$$p_F = h\left(\frac{3}{8\pi}\right)^{1/3} n_e^{1/3}$$

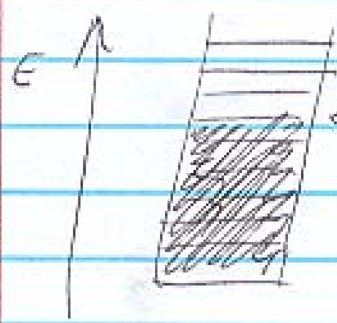
density of free electrons

Recall from SR:

$$E^2 = m_0^2c^4 + p^2c^2 \overset{UR}{\doteq} p^2c^2$$

$$\Rightarrow \quad E = pc$$

<u>Define</u>: Fermi energy $\epsilon_F = p_F c$



$\leftarrow \epsilon_F > 1.3 \text{ MeV} = (m_n - m_p)c^2$

Neutron decay is prohibited
("no space in the Fermi sea!")

<u>Furthermore</u>: it is even favorable to get rid of free e^-!

$$e^- + p^+ \longrightarrow n + \nu_e$$

(inverse β-decay; electron capture)

→ matter becomes increasingly neutron-rich

→ "neutronization"

- Neutron degeneracy pressure

n's become degenerate for $\rho \gtrsim 10^{16}\ \mathrm{kg/m^3}$

⇒ (NR) degeneracy pressure

$$P = \frac{h^2}{5 m_0} \left(\frac{3}{8\pi}\right)^{2/3} n^{5/3}$$

- Here: $m_0 = m_p \simeq m_n \simeq m_H = 1.67 \times 10^{-27}\ \mathrm{kg}$

$n = n_n \doteq$ neutron number density

$$n_n = \frac{\rho}{m_H} \quad \text{for NS}$$

n's become relativistically degenerate when:

$$m_H c^2 \simeq p_F c \simeq h\, n_n^{1/3} c$$

$$\Rightarrow \rho \simeq 10^{17}\ \mathrm{kg/m^3}$$

• <u>Upper mass limit for a NS</u>

<u>Recall</u>: $\frac{R_s}{R} = \frac{2GM}{c^2 R} = 0.5$ for NS

$\longrightarrow$ GR is crucial

Describe the mechanical structure of a NS with the OV equation:

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2}\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi r^3 P}{mc^2}\right)\left(1 - \frac{2Gm}{c^2 r}\right)^{-1}$$

Consider idealized case ($\rho = \rho_0$ = constant)

$\rightarrow$ "Incompressible gas"

$\longrightarrow m = m(r) = \frac{4\pi}{3} r^3 \rho_0$; $M = M(R) = \frac{4\pi}{3} R^3 \rho_0$

total mass

<u>Separation of variables</u>:

$$\int_{P(r)}^{P(R)=0} \frac{dP}{\left(1 + \frac{P}{\rho_0 c^2}\right)\left(1 + \frac{3P}{\rho_0 c^2}\right)} = -\frac{4\pi}{3} G \rho_0^2 \int_r^R \left(\frac{r\,dr}{1 - \frac{8\pi G \rho_0}{3c^2} r^2}\right)$$

$$\Rightarrow \frac{-1}{2}\rho_0 c^2 \left[\ln\left(\frac{1+\frac{p}{\rho_0 c^2}}{1+\frac{3p}{\rho_0 c^2}}\right)\right]_{p(r)}^{0}$$

$$= \frac{1}{4}\rho_0 c^2 \left[\ln\left(1 - \frac{8\pi G\rho_0}{3c^2} r^2\right)\right]_r^R$$

$$\Rightarrow \frac{1+\frac{p}{\rho_0 c^2}}{1+\frac{3p}{\rho_0 c^2}} = \frac{\left(1-\frac{2GM}{c^2 R}\right)^{1/2}}{\left(1-\frac{2GM}{c^2 R}\left(\frac{r^2}{R^2}\right)\right)^{1/2}}$$

$$\boxed{p(r) = \rho_0 c^2 \frac{\sqrt{1-\frac{R_s r^2}{R^3}} - \sqrt{1-\frac{R_s}{R}}}{3\sqrt{1-\frac{R_s}{R}} - \sqrt{1-\frac{R_s r^2}{R^3}}}, \quad \text{when } R_s = \frac{2GM}{c^2}}$$

Evaluate central pressure:

$$p_c = p(r=0) = \rho_0 c^2 \frac{1-\sqrt{1-\frac{R_s}{R}}}{3\sqrt{1-\frac{R_s}{R}} - 1}$$

Compare with Newtonian pressure at the center :

$$P_c = \frac{2\pi G \rho_0^2}{3} R^2$$

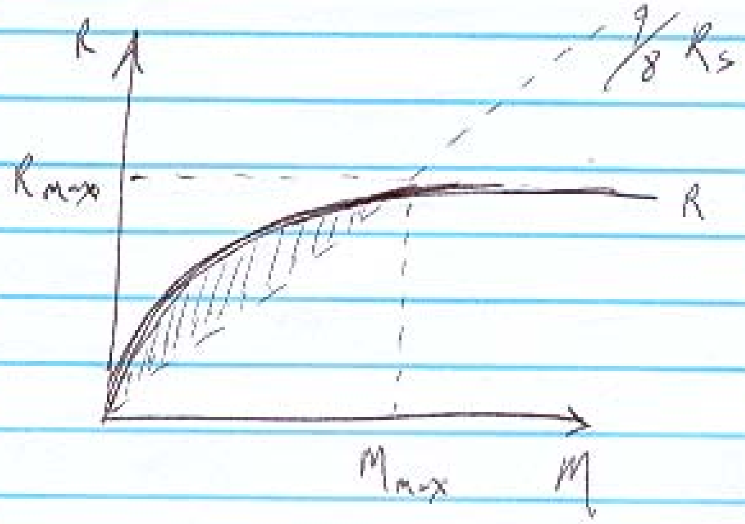
<u>Notice</u> : $P_c \longrightarrow \infty$, as $R \rightarrow \frac{9}{8} R_s$

What does this mean?

$\longrightarrow R_{NS} > \frac{9}{8} R_s$ (No infinite pressures in nature)

(1) $\frac{9}{8} R_s = \frac{9}{4} \frac{GM}{c^2}$

(2) $R = \left(\frac{3}{4\pi \rho_0} \right)^{1/3} M^{1/3}$



Solve for M_{max}:

$$\frac{9}{4}\frac{G}{c^2}M_{max} = \left(\frac{3}{4\pi\rho_0}\right)^{1/3} M_{max}^{1/3}$$

$$\boxed{M_{OV} \equiv M_{max} = \frac{8}{27}\left(\frac{c^2}{G}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}}$$

"Oppenheimer - Volkoff limit", $\rho_0 \simeq 5 \times 10^{17}\ kg\,m^{-3}$

upper mass limit for a neutron star $\Rightarrow M_{OV} \simeq 5\ M_\odot$