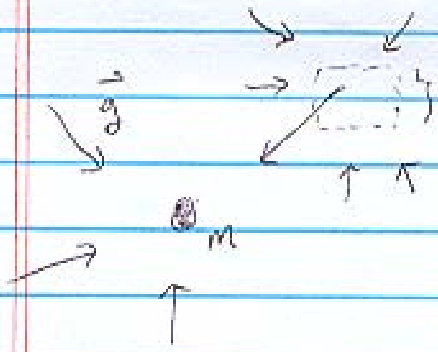
3/20/07

General Relativity (final)

- Very complicated! $\rightarrow$ Non-linear!



energy density from gravitational field incites its own gravitational effects

Hydrostatic equilibrium in GR

- generalize Newtonian expression $\dfrac{dP}{dr} = -\dfrac{Gm}{r^2}\rho$

(1) $\rho \hat{=}$ "measure of inertia" $\rightarrow$ inertial mass density

- in relativity

total energy density, $\rho c^2 + P$

ρ, P

$\rightarrow$ Replace $\rho \rightarrow \rho + \dfrac{P}{c^2}$

(2) $m \hat{=}$ "source of gravity"

$\rightarrow$ Replace $m \rightarrow m_{eff} = \dfrac{4\pi}{3} r^3 \rho_{eff} = \dfrac{4\pi}{3} r^3 \rho + 4\pi r^3 \dfrac{P}{c^2}$

$$\Rightarrow m_{eff} = m + 4\pi r^3 \frac{P}{c^2}$$

$$= m\left(1 + \frac{4\pi r^3 P}{mc^2}\right)$$

(3) Space is curved in a gravitational field

$\rightarrow$ replace $r^2 \rightarrow r^2\left(1 - \frac{2GM}{c^2 r}\right)$

Now,

$$\frac{dP}{dr} = -\rho\left(1+\frac{P}{\rho c^2}\right)\frac{GM\left(1+\frac{4\pi r^3 P}{mc^2}\right)}{r^2\left(1-\frac{2GM}{rc^2}\right)}$$

$$= -\rho\frac{GM}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2GM}{rc^2}\right)^{-1}$$

"Oppenheimer - Volkoff (OV) equation"

Notice: • You can recover the Newtonian form of this equation if $c \rightarrow \infty$

$\rightarrow$ Newton's theory is "action at a distance"

• You can recover Newton's form also for a weak gravitational field:

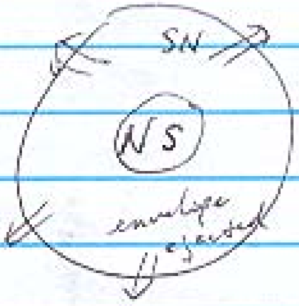
$$\frac{GM}{c^2 r} \ll 1 \quad \text{and} \quad P \ll \rho c^2$$

⇒ OV equation describes structure of neutron stars!

Neutron Stars

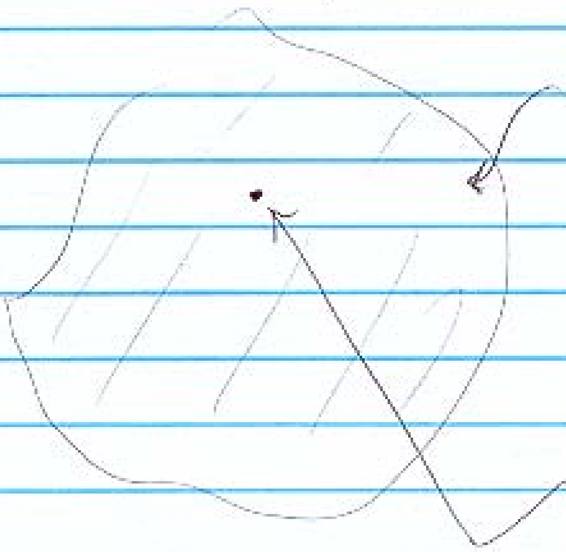
- Discovery: first postulated by theory (Baade & Zwicky) ↳ 1934 –
  - Idea (now known to be correct): (Landau, 1938)

    "Neutron stars (NS) are born when massive stars die in a supernova (SN) explosion
    → Initial mass of star $\gtrsim 8\,M_\odot$



  - By serendipity, radio astronomers observe pulsars (Bell & Hewish 1967)

e.g. in Crab nebula



synchrotron radiation (powered by pulsar

pulsar = rotating NS

→ Can not be WDs!

→ Rotation would have torn a WD apart

→ Oscillations too fast for WD pulsational period:
$\sim \tau_{ff_{WD}} \sim 1$ sec

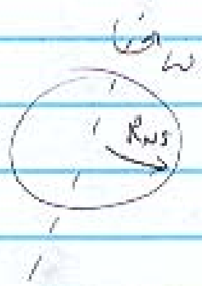
Basic Properties

→ pulsar observations

- $M_{NS} \sim 2\, M_\odot$ (from binary pulsars)

- Estimate radius from observed pulsation period:

$P_{NS} = 1/30$ sec



centrifugal acceleration < gravity

rotation speed at surface: $V_{rot} \simeq \frac{2\pi R_{NS}}{P_{NS}}$

$$\frac{V_{rot}^2}{R_{NS}} < \frac{G M_{NS}}{R_{NS}^2} \Rightarrow R_{NS} < \left(\frac{G M_{NS} P_{NS}^2}{4\pi^2}\right)^{1/3}$$

$$\simeq 100 \text{ km}$$

Now known that $R_{NS} \sim 10$ km

Average density $\langle \rho \rangle_{NS} = \dfrac{M_{NS}}{\left(\frac{4\pi R_{NS}^3}{3}\right)} \sim 10^{17} \ kg \ m^{-3}$

For comparison, $\langle \rho \rangle_{\odot} \simeq 1{,}000 \ kg \ m^{-3}$

$\langle \rho \rangle_{WD} = 10^9 \ kg \ m^{-3}$

An extreme density!

→ Nuclear density $\sim \dfrac{m_H}{\frac{4\pi}{3} r_0^3}$

→ Nuclear radius $R = r_0 \underbrace{A}_{\text{nucleon number } (n+p)}^{1/3}$

$r_0 = 1$ fermi $= 10^{-15}$ m

→ NS are like giant atomic nuclei, but:

- NS is held together by gravity
- Nucleus is held together by strong force