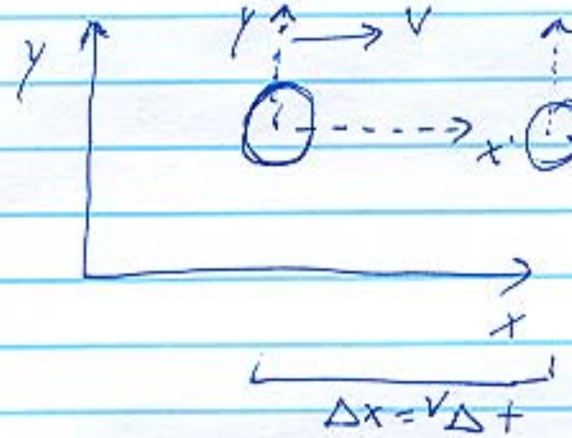
2/27/07

General Relativity (Very brief intro.)

• Special Relativity (SR)

- Example: time dilation



→ in clock rest frame, $\Delta x' = \Delta y' = \Delta z' = 0$

$\Delta t' = \tau \doteq$ proper time

is "absolute"

$$\Delta t' = \frac{\Delta t - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\Delta t - \frac{v}{c^2}\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \Delta t\sqrt{1-\frac{v^2}{c^2}}$$

$$\Longrightarrow \Delta t' \le \Delta t$$

"Moving clocks move slower"

- time and space are relative

Q: What is 'real' in (SR)?

Minkowski's BIG idea (1908):

$$(\Delta x')^2 = \frac{(\Delta x - v\,\Delta t)^2}{1 - \frac{v^2}{c^2}} \qquad (\Delta y')^2 = (\Delta y)^2$$

$$(\Delta z')^2 = (\Delta z)^2$$

$$(\Delta t')^2 = \frac{\left(\Delta t - \frac{xv}{c^2}\right)^2}{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - (c\,\Delta t')^2$$

$$= (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\,\Delta t)^2$$

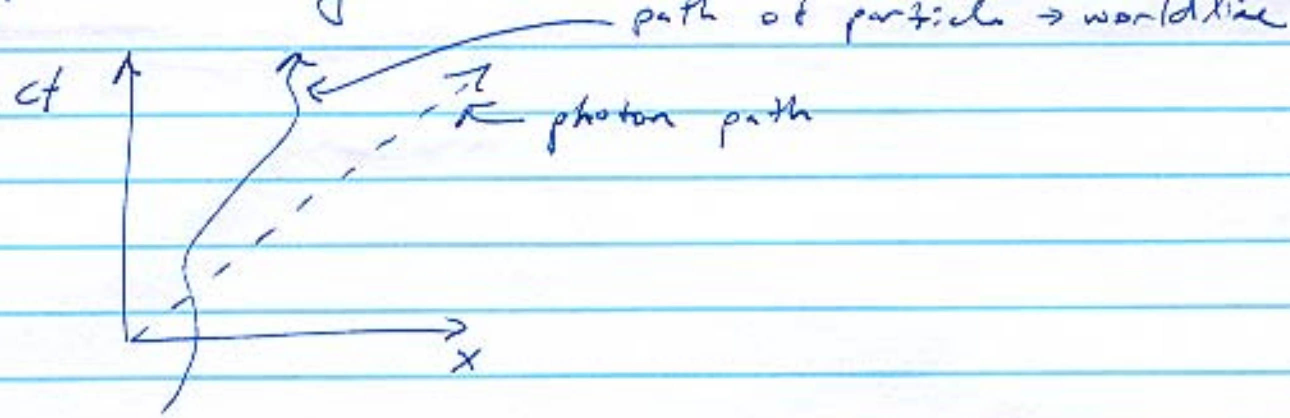
"Spacetime interval" is absolute, i.e. all observers measure this the same

$$ds^2 = -c^2dt^2 + dx^2 + \underbrace{dy^2 + dz^2}_{\text{often zero}}$$

→ Much like Pythagorean thm. in 3D space

Q: What is spacetime?
- 4-dimensional (t, x, y, z)
- real arena for all of physics
- collection of events ('manifold')

- Spacetime diagram



- Characterize geometry of spacetime in neat way:

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

$$= \sum_{\mu\nu} \eta_{\mu\nu}\, dx^\mu dx^\nu \qquad \mu = 0,1,2,3 \quad \nu = 0,1,2,3$$

↳ $dx^0 = c\,dt$

$$\eta_{\mu\nu} = \underbrace{\begin{array}{c|c} & \nu \rightarrow \\ \hline \mu \downarrow & \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}}_{\text{"Minkowski metric"}}$$

Write even neater: $ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu$

"Einstein summation rule": sum over repeated indices

→ spacetime in SR is 'flat'

Q: What do we mean by this?

A: Metric coefficients ($\eta_{\mu\nu}$) do not depend on time or space, i.e.

$$\boxed{\frac{\partial \eta_{\mu\nu}}{\partial x^\alpha} = 0}$$

4-vector in spacetime, e.g.

$x^\mu = (\overset{0}{ct}, \overset{1}{x}, \overset{2}{y}, \overset{3}{z})$ "4-position"

$p^\mu = \left(\frac{E}{c}, p_x, p_y, p_z\right)$ "4-momentum"

<u>Equation of motion</u>

<u>Newton</u> vs. <u>Einstein</u> (SR)

$m \frac{d^2 \vec{r}}{dt^2} = \vec{F}$ $\qquad\qquad$ $m_0 \frac{d^2 x^\mu}{d\tau^2} = f^\mu$

↑ 4-force

$m \longrightarrow m_0$

$dt \longrightarrow d\tau$

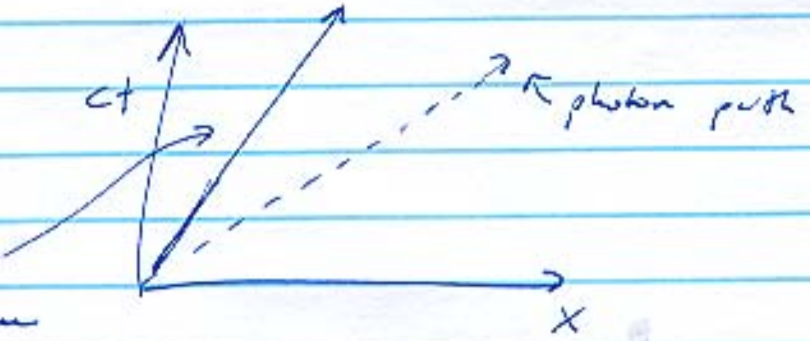
3-vectors $\longrightarrow$ 4-vectors

$(\vec{r} \longrightarrow x^\mu)$

e.g. force-free motion

$$\frac{d^2 x^\mu}{d\tau^2} = 0$$

$\Rightarrow$ straight-line motion in spacetime



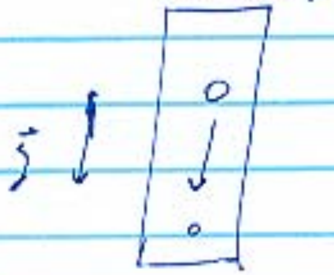
General Relativity

Basic idea: What is gravity?
- Einstein's answer

Part 1: "Gravity can always be transformed away!"

- consider elevator

at rest

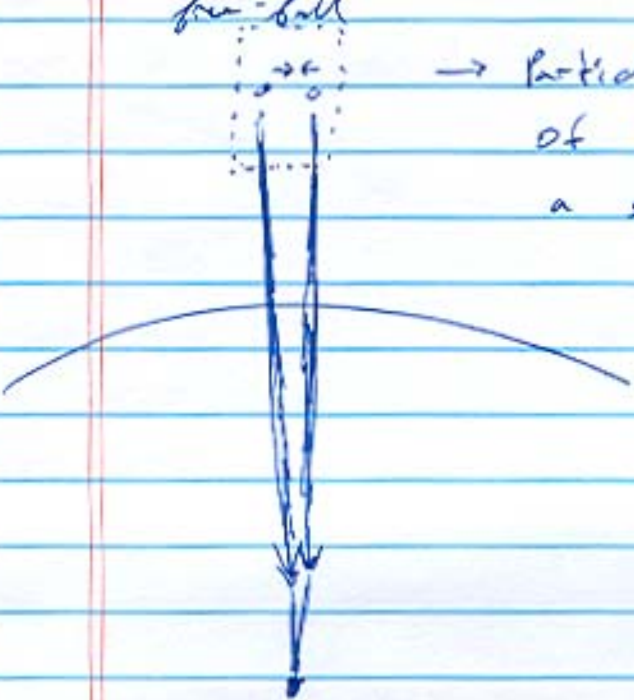


free-fall

In free-fall frame: Laws of physics are the same as if gravity were absent

"Equivalence Principle"

part 2: But: one manifestation of gravity that can not be made to disappear!



→ Particles converge, although each of them individually falls along a straight line!

→ "Tidal forces"

→ Tidal effect is akin to behavior of straight lines on curved surfaces!

e.g. surface of Earth

Einstein's GREAT idea

"gravity is the curvature of spacetime"