2/22/07

General Relativity: Very Brief Introduction

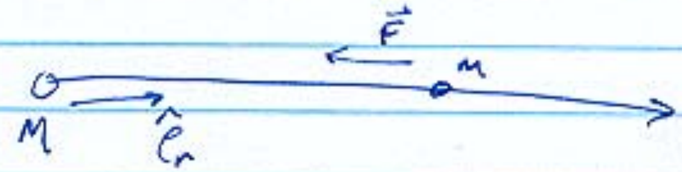
What is General Relativity (GR)?

- Gravity + Special Relativity (SR) $\rightarrow$ GR

- Basic idea of GR:
  Gravity is the curvature of spacetime

Recall Newton's theory of gravity:

- Consider force between two particles



$$\boxed{\hat{e}_r = \frac{\vec{r}}{|\vec{r}|}}$$

$\uparrow$ unit vector

$$\vec{F} = -\frac{GMm}{r^2}\hat{e}_r$$

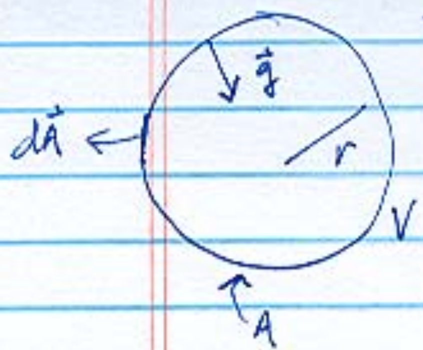
or $\quad \vec{F} = m\vec{g} \quad , \quad m = m_g =$ "gravitational mass"

Now, gravitational potential ("field")

$$\longrightarrow \quad \varphi = -\frac{GM}{r}$$

$$\Longrightarrow \quad \vec{g} = -\nabla\varphi$$

Rephrase this:



$$-\int \nabla^2 \varphi \, dV = \int \nabla \cdot \vec{g} \, dV \underset{\text{Gauss}}{=} \oint_A \vec{g} \cdot d\vec{A}$$

$$\boxed{\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}$$

$$\oint_A \vec{g} \cdot d\vec{A} = -4\pi r^2 g$$

$$= -4\pi G \underbrace{\int_V \rho(r) \, dV}$$

Here, $M = \int_V \rho(r) \, dV$

$$\Longrightarrow \boxed{\nabla^2 \varphi = 4\pi G \rho}$$ "The Poisson Equation"

→ The field equation in Newton's theory

<u>Equation of motion</u>

$$m_i \frac{d^2\vec{r}}{dt^2} = \vec{F} = m_g \vec{g}, \quad m_i = \text{"inertial mass"}$$

Empirical fact (Galileo → Pisa)

$$\frac{d^2\vec{r}}{dt^2} = \frac{m_g}{m_i}\,\vec{g} = \text{constant}$$

$$\Rightarrow \frac{m_g}{m_i} = \text{constant} \Rightarrow m_g = k m_i$$, by convention $k = 1$.

$$\Rightarrow \boxed{\frac{d^2\vec{r}}{dt^2} = \vec{g} = -\nabla\varphi}$$

Gravitational Redshift

- Use Newton's theory (no GR as of yet)

$$\epsilon_\infty = \frac{hc}{\lambda_\infty}$$

$$\epsilon_{em} = h\nu_{em} = \frac{hc}{\lambda_{em}} \Rightarrow \boxed{m_{ph} = \frac{\epsilon_{em}}{c^2}}$$?

Photon has to climb out of potential well

- work done: $W = -\int_R^\infty \vec{F}\cdot d\vec{r}$

$$= \frac{GMm_{ph}}{R}$$

$$= \frac{GM}{Rc^2}\,\epsilon_{em}$$

$$\epsilon_\infty = \epsilon_{em} - \frac{GM}{Rc^2}\epsilon_{em} = \epsilon_{em}\left(1 - \frac{GM}{Rc^2}\right)$$

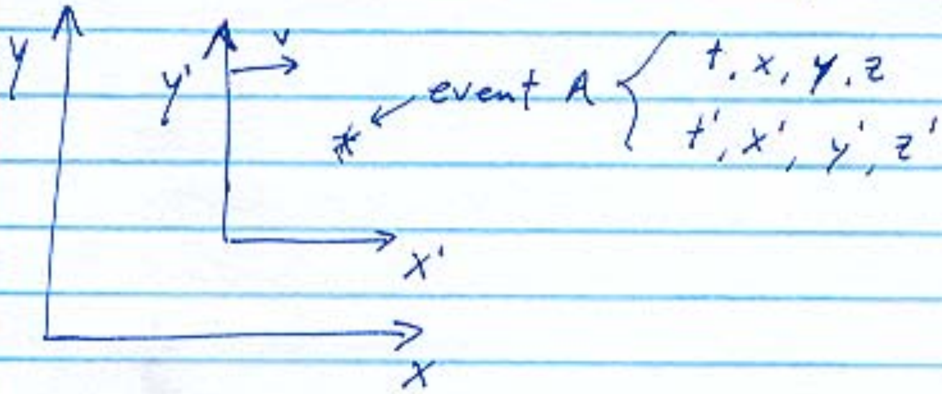
$$\Rightarrow \boxed{\nu_\infty = \nu_{em}\left(1 - \frac{GM}{Rc^2}\right)} \text{ , or}$$

$$\nu_\infty = \nu_{em}\left(1 + \frac{\varphi}{c^2}\right) \quad \text{with} \quad \varphi = \frac{-GM}{R}.$$

$\Rightarrow$ Also approximately true in Einstein's theory

Special Relativity (SR)

- Consider events in different inertial frames
  - Event specified by coordinates in time and space : (t, x, y, z)



Lorentz transformations

$$y' = y$$
$$z' = z$$
$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - v^2/c^2}}$$

Consider differences between two events A, B

$$\Delta x = x_B - x_A \longrightarrow d_x$$

$$\Delta t = t_B - t_A \longrightarrow dt$$