2/20/07

White Dwarfs (con't)

Rephrase the Chandrasekhar mass limit for a white dwarf in terms of the 'Planck mass,'

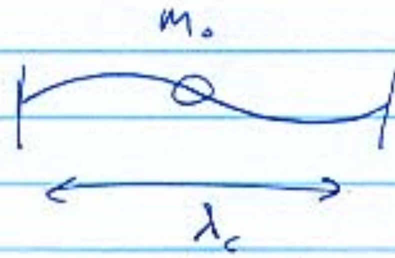
$$M_{Pl} = \left(\frac{hc}{G}\right)^{1/2} = 5 \times 10^{-5}\,g$$

$$= 10^{19}\ \frac{GeV}{c^2}$$

meaning: mass of the smallest black hole that can be described by GR

Compton wavelength, a fundamental distance, λ_c, from a mass at rest

$$\epsilon = m_o c^2 = \frac{hc}{\lambda_c}$$



For a black hole to be well-defined, need

$$GR \rightarrow R_s \gtrsim \lambda_c \leftarrow QM$$

$$\Rightarrow \frac{GM_{BH}}{c^2} \gtrsim \frac{h}{M_{BH}c}$$

$$\Rightarrow \boxed{M_{BH} \gtrsim \left(\frac{hc}{G}\right)^{1/2}}$$

M_{Pl}

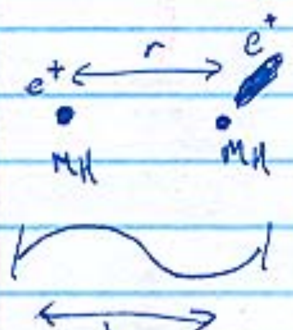
At the Planck scale, we need quantum gravity to describe nature (GR + QM)

$$\Longrightarrow M_{ch} = \frac{1}{4}\left(\frac{3}{8}\right)^{3/2} \frac{m_{Pl}^3}{m_H^2}$$

$$\Longrightarrow \boxed{M_{ch} \simeq \frac{m_{Pl}^3}{m_H^2}}$$

Now, rephrase M_{ch} in terms of the 'strength of gravity':

Recall how to measure strength of an E·M interaction:



$\Longrightarrow$ potential energy

$$U_{em} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}.$$

Evaluate U_{em} at $r = \lambda_c$. This defines the "fine structure constant":

$$\alpha = \frac{U_{em}(r=\lambda_c)}{m_H c^2} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\frac{\hbar}{m_H c} m_H c^2}$$

$$\simeq \frac{e^2}{4\pi\epsilon_0 \hbar c} \simeq \frac{1}{137}$$

By analogy, define the "gravitational fine structure constant":

$$\alpha_G \equiv \frac{|U_G(r=\lambda_c)|}{m_H c^2} = \frac{G m_H^2}{\hbar c} \simeq 10^{-38},$$

much smaller than α, i.e. gravity is very weak compared to electromagnetism.

Now, $$M_{ch} \simeq \left(\frac{\hbar c}{G}\right)^{3/2} \frac{1}{m_H^2} = \left(\frac{\hbar c}{G m_H^2}\right)^{3/2} m_H$$

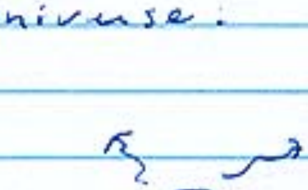
$$\simeq \alpha_G^{-3/2} m_H$$

$$= 10^{57} m_H \sim 1 M_\odot$$

Since gravity is so weak we need $\sim 10^{57}$ nucleons to overpower the e-m interaction

$\Rightarrow M_{ch} \sim 1 M_\odot$, the typical mass of any star in our universe!

Cooling of White Dwarfs



$L = \frac{\text{energy}}{\text{time}}$

R - radius

T - surface temperature

$$L = 4\pi R^2 \sigma_{SB} T_{eff}^4 \quad \text{(Stefan - Boltzmann law)}$$

$$\sigma_{SB} = 5.67 \times 10^{-8}\ W\, m^{-2}\, K^{-4}\ .$$

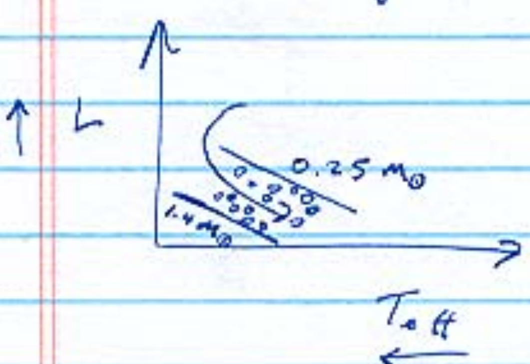
- Mass - radius relation for a NR WD :

$$R = 0.01\, R_\odot \left(\frac{M}{M_\odot}\right)^{-1/3}$$

From observations, we know that $T_{eff} \sim 20{,}000\,K$

$$\Longrightarrow L_{WD} = 10^{-2}\, L_\odot \left(\frac{M}{M_\odot}\right)^{-2/3} \left(\frac{T_{eff}}{20{,}000\,K}\right)^4$$

Consider cooling tracks of WD's in Hertzsprung - Russel diagram :



Q: How long does it take for a WD to lose all of its thermal energy, E_{th} ?

A: Define the 'cooling time' :

$$\tau_{cool} \simeq \frac{E_{th}}{L_{WD}}\ , \quad L_{WD} \sim 10^{-2} L_\odot\ , \quad L_\odot \sim 4 \times 10^{26}\,W$$

$$E_{thermal} \simeq \frac{3}{2} k_B T \, \frac{M}{12 m_H}$$

For WD: $M \sim 1 M_\odot$

$T \sim 10^8 K$ $\quad (3He^4 \rightarrow C^{12})$

$\Longrightarrow E_{th} \sim 10^{41} J$

$\Longrightarrow \tau_{cool} \simeq 10^9 yr$ - comparable to the age of the Universe/Galaxy

$t_H \sim 10^{10} yr$